Optimal Monetary Policy with Signal Extraction

Shangdi Hou*

December 5, 2022

Abstract

In this paper we study the optimal discretionary monetary policy under partial information (PI) where the central bank can only extract information from an endogenous signal, price inflation. The signal is determined in equilibrium by the policy rate and the unobserved supply and demand shocks. We solve for optimal policy in a non-linear model where the Phillips curve is bent by asymmetric wage adjustment costs and the "certainty equivalence" principal that prevails in linear models cannot be applied. Optimal policy prescribes that the central bank should raise the interest rate gradually when price inflation is low but respond strongly when it is high. This non-linearity arises because signal extraction interacts differently with optimal policy depending on the price inflation observed.

Keywords: Optimal monetary policy, Partial information, Policy cautiousness

JEL Codes: E52, E58

*Universitat Autònoma de Barcelona & Barcelona School of Economics. E-mail: houshangdi@gmail.com.
I am specially grateful to my advisor Prof.Albert Marcet. I would also like to thank Jordi Caballé, Davide Debortoli, Min Fang, Alexander Ludwig, Francesc Obiols-Homs, Shengliang Ou, Hugo Rodríguez, Luis Rojas, Donghai Zhang, Renbin Zhang, Tongbin Zhang, participants of Bellaterra Macro Club, BSE PhD Jamboree, macro seminar at Shandong University for their comments and help. All errors are my own.

1 Introduction

In the recent studies of monetary policy, simple policy rules such as Taylor rules could achieve good results in simulated small macroeconomic models. At the same time, many empirical studies report that the policy specifications of this kind fit the actual behavior of the central banks in several countries.

But the central banks actually face much more complicated situation compared with the set-ups in the model. In particular, they do not know the underlying state of the economy in real time but only infer it from limited data set. This poses a challenge to the central bank, the information constraint. This challenge is twofold: how easy it is to get the relevant data and how accurate the indicators could reflect the fundamentals of the economy. For example, the central bank can only have very preliminary measure of current period GDP and update it at least one period later. Another key indicator for monetary policy, wage inflation, is difficult to observe because it is kept within employees and firms while the central bank can only rely on the survey data which is subject to large measurement error. Even though some indicators are much easier to get with accuracy, the central bank may still face great uncertainty when making monetary policy. A typical example is price inflation, which can be driven by a high demand shock or by a supply shortage or a combination of both. Different causes needs to be treated differently. The haziness would not disappear even though the central bank knows the price inflation perfectly. So the typical presumption in the macroeconomic models that policymakers know the state of the system at a point in time generally does not hold in policy field, policy decisions need to be made under partial information (PI) instead of full information (FI).

Apart from information barriers mentioned above, the central bank also needs to take into account that the signals it observes are endogenous to the policy rule. Indicators like inflation, output are determined not only by the fundamentals in the economy, but also by the monetary policy the central bank adopts. So the policy optimization and the signal extraction need to be solved simultaneously.

To our knowledge, all existing results in the literature on optimal monetary policy with PI circumvent this technical issue of simultaneity by introducing timing assumptions such that signal extraction and policy optimization can be solved separately. While this literature has led to many interesting applications, it can say nothing about how policy optimization and signal extraction interact with each other. What is emphasized in the literature is policy optimization under PI or uncertainty but what is missing is how policy choice affects the uncertainty the central bank faces. Svensson and Woodford (2004) address this simultaneity problem in linear model and develop a method called "certainty equivalence", which allow them to solve the signal extraction and optimal choice problems sequentially. But their method cannot be applied to many important non-linear scenarios in monetary policy analysis, such as zero lower bound, models with financial frictions etc.

In this paper, we focus on the non-linearity of new Keynessian Phillips curve, the substantial curvature in the relationship between money wage growth and unemployment. Phillips (1958) conjectures that this curvature owed to the fact that "... workers are reluctant to offer their services at less than prevailing rates when the demand for labour is low and unemployment is high so that wage rates fall only very slowly." As is supported empirically and modelled theoretically in the literature, We study the optimal discretionary monetary policy with signal extraction in such an economy where the Phillips curve is bent by the asymmetric wage adjustment cost. To the best of our knowledge, we are the first to study the optimal monetary policy with signal extraction in non-linear models.

To make the model more transparent and tractable, we assume the only nominal rigidity is asymmetric wage adjustment cost. When the central bank has full information on the economy, the optimal policy calls for strict wage inflation targeting and full stabilization of demand shocks. We introduce the information constraint through an identification problem: every period the economy is hit by two exogenous shocks, the supply and demand shocks, but the central bank can only infer the state of the economy from one single indicator, price inflation. We find that the responding rule of nominal rate to price inflation is quite non-linear: the central bank should raise the nominal rate gradually when price inflation is low but raise it sharply when price inflation is high. We argue that this non-linearity arises because the signal extraction problem interacts differently with optimal monetary policy depending the range of price inflation. In particular, the sluggish adjustment around the low realization of price inflation is justified by the strong real effects of monetary policy as small change of policy rate can make great influence on output. The inertial behaviour in the intermediate range of price inflation can be seen as policy cautiousness as the central bank faces more uncertainty in this regime. The strong response to high levels of price inflation is a choice under less uncertainty and the fading real effects of monetary policy.

Our model provides a plausible explanation of the Fed's monetary policy after COVID-19. When the inflation rate is below 3 per cent, the Fed was not sure about the cause of inflation, a supply shock or a demand shock. So it should move cautiously in such haziness. On top of that, the Fed also believes that the monetary policy is very powerful in inflation controlling as the Phillips curve is flat in this interval. But after January 2022, the inflation rate grew even higher, the Fed is more certain about the true cause behind it, the demand shock, so it raised the interest rates without hesitation, considering also the steep Phillips curve in this interval.

To highlight our contribution relative to the literature, we compare our results with some alternative policy choices and show that the endogeneity of the signal and the non-linearity of the wage Phillips curve bent by asymmetric wage adjustment cost are vital in monetary policy making and ignoring that could lead to great welfare loss.

The remainder of the paper is organized as follows. We discuss the related literature in Section 2. Section 3 introduces our main model and the solution under full information. In Section 4 we present the result and interpretation of the optimal monetary policy under partial information. Section 7 compares the optimal policy with some alternative policy rules and their welfare implications. For completeness, we present the solution for the case of serial correlated shocks in Section 8. Section 9 concludes.

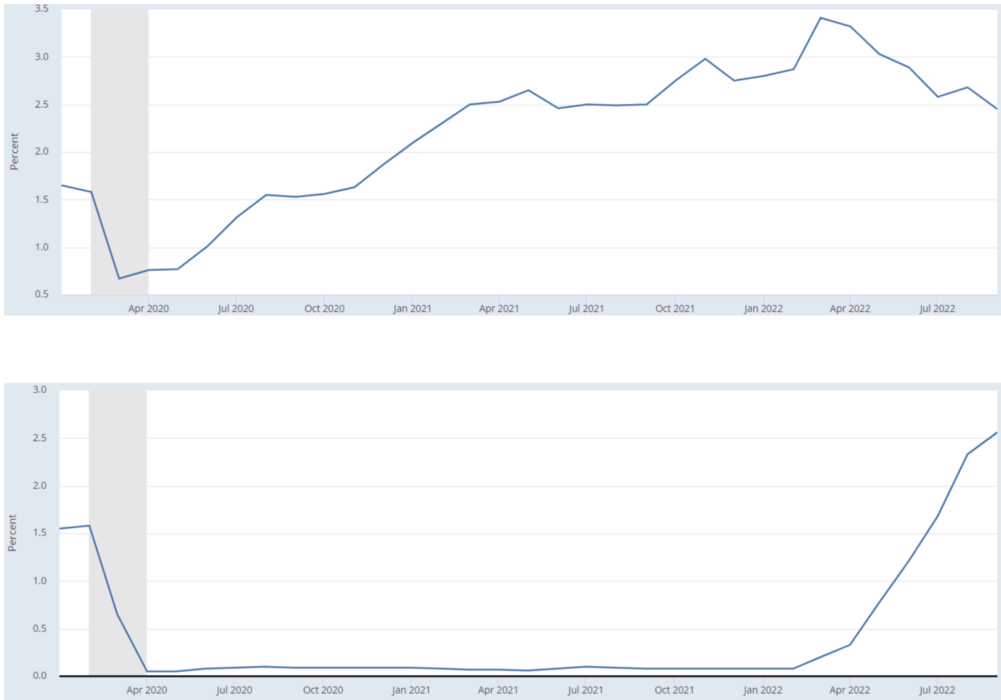


Figure 1: U.S. Inflation and Policy Rates

2 Literature Review

Optimal monetary policy with signal extraction is often considered in linear models: Svensson and Woodford (2004) shows that in the case of a linear economic model with a quadratic welfare loss function, a principle of "certainty equivalence" applies: the government applies the policy under full information to its best estimate of the state of the economy. Aoki (2003) applies their results to optimal monetary policy with noisy indicators on output and inflation. Nimark (2008b) applies them to a problem of monetary policy where the central bank uses data from the yield curve knowing that the chosen policy affects the very same data. Relatedly, Morris and Shin (2018) analyze the optimal weight on an endogenous signal in a linear policy rule.

But "certainty equivalence" cannot be applied to study the optimal monetary policy under partial information when the economy features some important non-linear relationships, a typical example of which is the substantial curvature in the relationship between money wage growth and unemployment. As documented by Phillips (1958), the curve is nearly vertical at high inflation and flattens out at low inflation, implying progressively larger output costs of reducing inflation. He conjectures that this curvature owed to the fact that "... workers are reluctant to offer their services at less than prevailing rates when the demand for Labour is low and unemployment is high so that wage rates fall only very slowly." The empirical evidence of downward nominal wage rigidity is provided in Akerlof et al. (1996) and Daly and Hobijn (2014). Kim and Ruge-Murcia (2009) and Benigno and Ricci (2014) study the optimal monetary policy in a dynamic stochastic general equilibrium model and find that the optimal inflation rate is positive. Fahr and Smets (2011) consider both nominal and real downward wage rigidity (DWR) in a monetary union and find that optimal grease inflation may be dampened by heterogeneity in the types of DWR in different regions.

We revisit Phillips' hypothesis that downward nominal wage rigidities bend the Phillips curve and consider how this non-linearity interacts with signal extraction under partial information. To the best of our knowledge, our paper is the first one to consider the optimal monetary policy with signal extraction in a non-linear model. The solution method to our model is based on the work of Hauk, Lanteri & Marcet (2021, HLM hereafter), which address the optimal fiscal policy with signal extraction problem from the first principal.

3 The structure of the economy

The model developed in this section is a small-scale, dynamic stochastic general equilibrium model with downward nominal wage rigidity.

3.1 Firms

Firms operate in a perfectly competitive goods market and produce output using the production function

$$Y_t = A_t N_t \tag{1}$$

Here A_t is average labor productivity, which evolves exogenously. The labor aggregate, N_t, is a Dixit–Stiglitz(1977) aggregate over a continuum of labor types $j \in [0,1]$ and is of the form

$$N_t = \left(\int_0^1 N_t(j)^{1-\frac{1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}} \tag{2}$$

where $N_t(j)$ is the quantity of type-j labor employed by the firm in period t. The parameter ϵ represents the elasticity of substitution among labor varieties.

Let $W_t(j)$ denote the nominal wage for type-j labor prevailing in period t, for all $j \in [0,1]$. As discussed below, nominal wages are set by workers of each type(or a union representing them) and taken as given by firms. Given the wages effective each period for different types of labor services, cost minimization by the firm yields the demand for each type of workers, given the firm's total employment N_t

$$N_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon} N_t \tag{3}$$

where

$$W_t = \left(\int_0^1 W_t(j)^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}} \tag{4}$$

is an aggregate wage index. Because firms operate in a perfectly competitive goods market, they set the goods price P_t equal to the marginal production cost,

$$P_t = \frac{W_t}{A_t} \tag{5}$$

which yields the relationship between price inflation and wage inflation:

$$\Pi_t^p = \Pi_t^w \frac{A_{t-1}}{A_t} \tag{6}$$

where $\Pi_{t+1}^p = \frac{P_{t+1}}{P_t}$ and $\Pi_t^w = \frac{W_{t+1}}{W_t}$ denotes price inflation and wage inflation respectively.

3.2 Households

The economy is populated by a large number of identical households. Each household is made up of a continuum of infinitely-lived members specializing in a different labor service and indexed by $j \in [0,1]$. Income is pooled within each household, which acts as risk sharing mechanism. The representative household chooses its path of consumption $\{C_t\}_{t=0}^{\infty}$ and wages and labor supply $\{W_t(j), N_t(j)\}_{t=0}^{\infty}$ to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t; Z_t) \tag{7}$$

and the utility function is defined by

$$U(C_t, N_t; Z_t) = Z_t \left(\ln C_t - \frac{\chi}{\eta + 1} \int_0^1 N_t(j)^{\eta+1} dj \right) \tag{8}$$

where $\chi \geq 0$ and $\eta \geq 0$ determine the dis-utility of labor supply. $\mathbb{E}_t$ is the expectation operator conditional on information at time t. $\beta \in (0,1)$ is the discount factor. The preference shock Z_t shifts overall utility level and disturbs the household's inter-temporal substitution of consumption. Households maximization problem is subject to a sequence of flow budget constraints, expressed in real terms as

$$C_t + \frac{B_t}{P_t} \leq \int_0^1 \frac{(1+\tau) W_t(j) N_t(j)}{P_t} dj - \int_0^1 \Phi(\frac{W_t(j)}{W_{t-1}(j)}) dj N_t + \frac{1 + i_{t-1}}{P_t} B_{t-1} + T_t \tag{9}$$

where B_t represents the quantity of one-period nominal riskless bonds purchased in period t and maturing in period $t+1$. The nominal interest paid during period t on the bonds held at the end of period $t-1$ is i_{t-1}. τ is an employment subsidy financed by means of lump-sum tax T_t that corrects the distortions caused by monopolistic competition in labor markets. τ is set to be equal to $\frac{1}{\epsilon - 1}$ so that the marginal rate of substitution between leisure and consumption equals to the real wage under flexible wage setting.

As monopolistic competitors, households choose their wage and labor supply taking as given the firm's demand for their labor type. Labor market frictions induce a cost in the adjustment of nominal wages. We assume the wage adjustment cost takes the form of an altered linex cost function similar to Varian (1974):

$$\Phi(\frac{W_t(j)}{W_{t-1}(j)}) = \frac{\phi-1}{2}(\frac{W_t(j)}{W_{t-1}(j)}-1)^2 + \frac{\exp(-\psi(\frac{W_t(j)}{W_{t-1}(j)}-1)) + \psi(\frac{W_t(j)}{W_{t-1}(j)}-1)-1}{\psi^2} \tag{10}$$

The parameter ϕ determines the degree of convexity and ψ the degree of asymmetry in adjustment costs around zero wage inflation(W_t/W_{t-1}=1). When $\psi > 0$, adjustment costs for nominal wage increases are smaller than those for nominal wage cuts of the same size, capturing asymmetries in nominal wage adjustments. The specification nests a quadratic function, $\lim_{\psi\to 0}\Phi(\frac{W_t(j)}{W_{t-1}(j)}) = \frac{\phi}{2}(\frac{W_t(j)}{W_{t-1}(j)}-1)^2$. Figure 2 gives a visual impression of a symmetric and asymmetric adjustment cost function. To simplify computations, we further

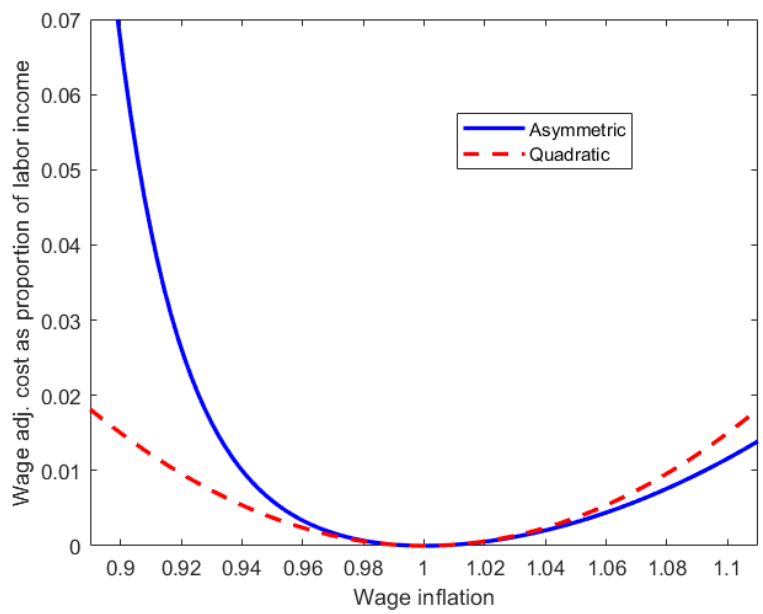


Figure 2: Adjustment cost functions

assume the labor adjustment cost is proportional to the aggregate employment N_t, instead of $\int_0^1 \Phi(\frac{W_t(j)}{W_{t-1}(j)})N_t(j)dj$. The households utility maximization yields the following optimality conditions:

$$\frac{Z_t}{C_t} = \beta\mathbb{E}_t\frac{Z_{t+1}(1+i_t)}{\Pi^p_{t+1}C_{t+1}} \tag{11}$$

$$\frac{\epsilon\chi(\eta+1)Z_tN_t(j)}{W_t(j)}+\frac{(1+\tau)(1-\epsilon)Z_t}{C_t}-\frac{Z_t\Phi'(\frac{W_t(j)}{W_{t-1}(j)})N_t}{C_tW_{t-1}(j)}+\beta\mathbb{E}_t\frac{Z_{t+1}\Phi'(\frac{W_{t+1}(j)}{W_t(j)})N_{t+1}W_{t+1}(j)}{C_{t+1}W_t^2}=0 \tag{12}$$

3.3 Symmetric Equilibrium

The model incorporates multiplicity of equilibria and we pick up the symmetric case, where all households supply exactly the same amount of labor and demand the same level of nominal wage, i.e. $W_t(j) = W_t$ and $N_t(j) = N_t$. Dropping the index j, the households optimality condition yields

$$\epsilon\chi Z_tN_t^\eta+\frac{(1+\tau)(1-\epsilon)W_tZ_t}{P_tC_t}-\frac{Z_t\Phi'(\Pi_t^w)\Pi_t^w}{C_t}+\beta\mathbb{E}_t\frac{Z_{t+1}\Phi'(\Pi_{t+1}^w)\Pi_{t+1}^wN_{t+1}}{C_{t+1}N_t}=0 \tag{13}$$

The economy-wide resource constraint is

$$C_t=(A_t-\Phi(\Pi_t^w))N_t \tag{14}$$

Combining them with $P_t = \frac{W_t}{A_t}$, one can get the wage Phillips Curve of the economy:

$$\frac{\epsilon\chi Z_tC_t^2}{[A_t-\Phi(\Pi_t^w)]^2}+\frac{(1+\tau)(1-\epsilon)A_tZ_t}{A_t-\Phi(\Pi_t^w)}-\frac{Z_t\Phi'(\Pi_t^w)\Pi_t^w}{A_t-\Phi(\Pi_t^w)}+\beta\mathbb{E}_t\frac{Z_{t+1}\Phi'(\Pi_{t+1}^w)\Pi_{t+1}^w}{A_{t+1}-\Phi(\Pi_{t+1}^w)}=0 \tag{15}$$

To illustrate how the downward nominal wage rigidity bends the Phillips curve, we plot the Phillips curve with different wage adjustment costs in the same figure. The point of zero inflation $\Pi^w = 1$ marked in Figure 3 represents the natural level of output and inflation. One can find that there is substantial curvature in the Phillips curve associated with asymmetric adjustment cost and it would be misleading to log linerize the model.

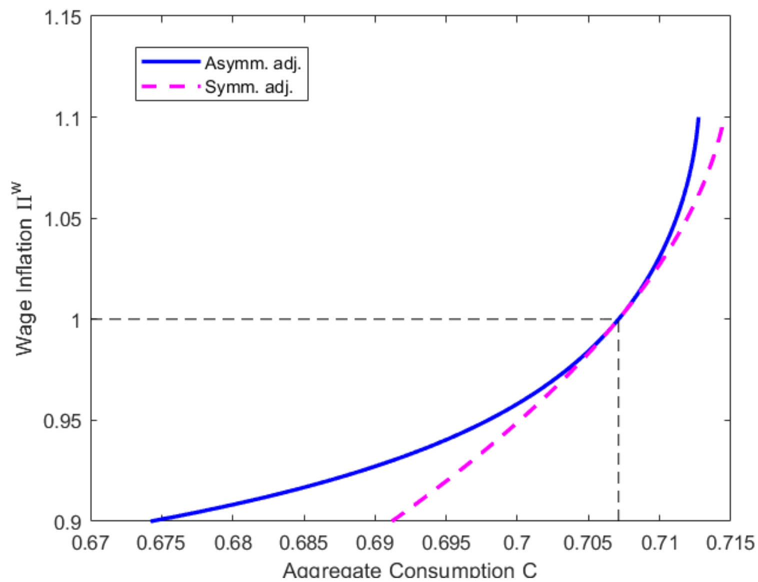


Figure 3: Wage Phillips Curve

3.4 Optimal discretionary monetary policy under full information

We consider the optimal monetary policy under discretion. In this case, the central bank cannot commit itself to any future action. The expectations in the Euler equation and the wage Phillips curve is taken as given by the monetary authority and will become a constant in equilibrium. We denote them as E and F respectively. Without loss of generality, we set A_{t-1} and Z_{t-1} equal to their steady state value 1. One can write the Euler equation and the wage Phillips curve as:

$$\frac{A_t Z_t}{C_t} = \beta(1 + i_t)E \tag{16}$$

$$\frac{\epsilon\chi Z_t C_t^2}{[A_t - \Phi(\Pi_t^w)]^2} + \frac{(1+\tau)(1-\epsilon)A_t Z_t}{A_t - \Phi(\Pi_t^w)} - \frac{Z_t \Phi'(\Pi_t^w)\Pi_t^w}{A_t - \Phi(\Pi_t^w)} + \beta F = 0 \tag{17}$$

Under this assumption the central bank's problem (18) becomes sequential optimization.

$$\max_{\{i_t, C_t, N_t, \Pi_t^w\}} U(C_t, N_t; Z_t) \tag{18}$$

$$\text{s.t. } (14), (16), (17)$$

choose the nominal interest rate i_t to maximize the household's utility, subject to the resources constraint, the dynamics IS curve and the wage Phillips curve.

The Lagrangian representation of the Ramsey problem is

$$
\begin{aligned}
\mathcal{L} =& U(C_t, N_t) + \lambda_t(C_t - (A_t - \Phi(\pi_t^w))N_t) \\
& + \mu_t(\beta(1+i_t)EC_t - A_t Z_t) \\
& + \nu_t(\epsilon\chi Z_t C_t^2 + (1+\tau)(1-\epsilon)A_t Z_t(A_t - \Phi(\Pi_t^w)) - Z_t\Phi'(\Pi_t^w)\Pi_t^w(A_t - \Phi(\Pi_t^w)) + \beta F(A_t - \Phi(\Pi_t^w))^2)
\end{aligned}
\tag{19}
$$

The first order necessary conditions associated with i_t is:

$$\mu_t \beta E C_t = 0 \tag{20}$$

and it is evident that $\mu_t = 0$. The intuition is that the central bank can always choose an interest rate level consistent with the Euler equation, in the absence of zero lower bound. The F.O.Cs associated with C_t, Π_t, N_t are:

$$[C_t] : U_C + \lambda_t + 2\nu_t \epsilon\chi Z_t C_t = 0 \tag{21}$$

$$[N_t] : U_N - \lambda_t \Phi(\Pi_t^w) = 0 \tag{22}$$

$$
\begin{aligned}
[\Pi_t^w] :& -\lambda_t N_t \Phi'(\Pi_t^w) - \nu_t[(1+\tau)(1-\epsilon)A_t Z_t \Phi'(\Pi_t^w) \\
& - Z_t\Phi'(\Pi_t^w)(1+\Pi_t^w)(A_t - \Phi(\Pi_t^w)) + Z_t\Pi_t^w\Phi'(\Pi_t^w)^2 - \beta F Z_t \Phi'(\Pi_t^w)(A_t - \Phi(\Pi_t^w))] = 0
\end{aligned}
\tag{23}
$$

These three F.O.Cs together with the resources constraint and the Phillips curve characterize the solution $C_t, N_t, \Pi_t^w, \nu_t, \lambda_t$ and the nominal rate can be solved from the Euler Equation. When the distortions caused by monopolistic competition is corrected through an employment subsidy, one cannot do better than to set $\Pi_t^w = 1$ and $i_t = \frac{Z_t}{\beta} - 1$ so that there are no losses to wage adjustment and the economy achieves its first best.

4 Optimal discretionary policy with signal extraction

The implementation of the optimal monetary policy requires the central bank to have accurate measure of demand shock or the wage inflation. But the former one cannot be observed directly from the data and the latter one can only rely on survey data, which is subject to large measurement error. Prices are public data and the central bank have much easier access to it. So a natural problem is to explore the optimal monetary policy contingent on price inflation. Now we assume that the only signal the central bank could observe is the price inflation and the monetary policy is a feedback rule which vary interest rates responding to the observed signal, the inflation.

4.1 Information structure and the timing

We first consider the case where the supply and demand shocks are i.i.d and uniformly distributed on a support $[A_{\min}, A_{\max}]$ and $[Z_{\min}, Z_{\max}]$ respectively. Suppose the central bank observes at time t the price level P_t, simultaneously with the choice of nominal interest rate i_t. The output Y_t cannot be observed in the current period but will be revealed next period. This implies the information set I_t of the central bank at time t is given by

$$I_t = \{P_t, i_t, Y_{t-1}\} \cup I_{t-1} \tag{24}$$

where $I_{t-1} = \{P_{t-1}, i_{t-1}, Y_{t-2}, P_{t-2}, i_{t-2}, Y_{t-3} \cdots\}$ and $I_0 = \{P_0, i_0, Y_{-1}, P_{-1}, i_{-1}\}$.

Given this information set, the central bank could identify the supply and demand shocks A_{t-1}, Z_{t-1} of the previous period, In order to focus our analysis on the implications of policy making with partial information, we abstract from any information constraint faced by private agents. They are assumed to have complete knowledge about the states of the economy, including the realization of supply and demand shocks, consumption and wage inflation. The justifications for this assumption are twofold, as pointed out by Aoki (2003): on the one hand, consumption and wages are the choice variable of the private agents, which is based on the agents' own preference and the firm's production capacity. On the other hand, consumption and production decisions of private sectors are not as dependent on the availability of aggregate data as is the policy decision of the central bank.

To be precise, we assume the following time sequence. At the beginning of time t, the output of the previous periods Y_{t-1} is revealed. Combining this with the information in I_{t-1}, the central bank is able to identify the true value of past supply and demand shocks A_{t-1} and Z_{t-1}. Then the central bank announces its policy rule

$$i_t = \mathcal{R}(\Pi_t^p) \tag{25}$$

according to a policy function $\mathcal{R} : \mathcal{P} \to \mathcal{T}$. $\mathcal{P}$ and $\mathcal{T}$ denote the set of possible policy rate and the price inflation observed, i.e. $\Pi_t \in \mathcal{P}$ and $i_t \in \mathcal{T}$. Once the supply and demand shock A_t, Z_t are realized, the economy arrivesits equilibrium.

Now the central bank cannot observe the fundamental shocks, A_t and Z_t in real time, but only infer the underlying state of the economy from the endogenous signal Π_t^P, taking into account that the policy variable i_t maps into the endogenous signal Π_t^P through the following reaction function

$$\Pi_t^p = h(i_t, A_t, Z_t) \tag{26}$$

which is defined as an implicit function from Euler equation (16) and wage Phillips curve (17):

$$\frac{A_t Z_t}{\beta(1+i_t)E} = \sqrt{\frac{A_t(A_t - \Phi(A_t\Pi_t^p))(\Phi'(A_t\Pi_t^p)\Pi_t^p - (1+\tau)(1-\epsilon))}{\epsilon\chi} - \frac{\beta F(A_t - \Phi(A_t\Pi_t^p))^2}{\epsilon\chi Z_t}} \tag{27}$$

The utility function can be expressed in terms of policy variables, endogenous signals and fundamental shocks as:

$$\mathcal{U}(i_t, \Pi_t^p, A_t, Z_t) = U\left(\frac{A_t Z_t}{\beta(1+i_t)E}, \frac{A_t Z_t}{\beta(1+i_t)(A_t - \Phi(A_t\Pi_t^p))E}; Z_t\right) \tag{28}$$

The central bank's optimization problem can be stated formally as:

$$\max_{\mathcal{R}:\mathcal{P}\to\mathcal{T}} \mathbb{E}[\mathcal{U}(i_t, \Pi_t^p, A_t, Z_t)] \tag{29}$$

$$\text{s.t. } (25), (26)$$

Notice that, mathematically, the only difference with the FI-problem (18) is the presence of constraint (25), which requires monetary policy to be a feedback rules which vary interest rate responding to price inflation Π_t^p.

4.2 Solution of optimal monetary under discretion

We denote the solution to the central bank's optimization problem (41) as $\mathcal{R}^*$. According to Hauk et al.(2021), $\mathcal{R}^*$ satisfies the following necessary optimality condition:

$$\int_{\mathbf{A}(\Pi^p,\mathcal{R}^*(\Pi^p))} \frac{\mathcal{U}_i^* + \mathcal{U}_\Pi^* h_i^*}{|h_Z^*|} f_Z(\mathcal{Z}^*(\Pi^p, a)) f_A(a) \mathrm{d}a = 0 \tag{30}$$

for almost all $a \in [A_{\min}, A_{\max}]$. Here $\mathbf{A}(\Pi^p, i)$ denotes the support of $[A_{\min}, A_{\max}]$ conditional on observing Π^p and i, i.e. $\mathbf{A}(\Pi^p, i) = \{a \mid \Pi = h(i, a, z)$ for some $a \in [A_{\min}, A_{\max}]$ and $z \in [Z_{\min}, Z_{\max}]\}$. $\mathcal{U}_i^*$, $\mathcal{U}_\Pi^*$, h_i^* denote the functions evaluated at the optimal solution. $\mathcal{Z}^*(\Pi^p, a)$ is a function satisfying $\Pi^p = h(\mathcal{R}^*(\Pi^p), \mathcal{Z}^*(\Pi^p, a), a)$.

Note that the value of the expectation E and F interacts with the optimal policy rule. To solve the model, we can apply the algorithm proposed in Hauk et al.(2021) with slight modification:Given the optimal policy under full information, $i^{FI}(A, Z) = \frac{Z}{\beta} - 1$, one can find the pairs (Π^p, i) for all possible realizations of (A, Z).

Algorithm 1 Then follow the following steps:

(1) Discretize the set of possible values for Π^p, which is $[\frac{1}{A_{max}}, \frac{1}{A_{min}}]$ in our model;

(2) Guess an initial value for E and F, denoted as E^0 and F^0;

(3) Taking $E = E^0$ and $F = F^0$, for each value $\overline{\Pi^p}$ on the grid created in step 1, one can

find the value i^* that solves the non-linear equation

$$\int_{\mathbf{A}(\overline{\Pi^p},i^*))} \frac{\mathcal{U}_i(i^*,(\overline{\Pi^p},a,z^*)+\mathcal{U}_\Pi(i^*,(\overline{\Pi^p},a,z^*)h_i(i^*,a,z^*)}{|h_Z(i^*,a,z^*)|} f_Z(z^*)f_A(a)\mathrm{d}a = 0 \qquad (31)$$

where z^* denote the value of z solving the equation $\overline{\Pi^p}-h(i^*,a,z)=0$ at a given a. $\mathbf{A}(\overline{\Pi^p},i^*))$ is the set of realizations of A with positive density given a pair $(\overline{\Pi^p},i^*)$;

(4) Given the policy rule found in step 3, one can find the updated value for E and F, denoted as E' and F'; If $|E-E^0|<\varepsilon$ and $|F-F^0|<\varepsilon$, where ε is the convergence criterion, stop; If not, take $E^0 \leftarrow E'$, $F^0 \leftarrow F'$ and go back to step (3).

5 Calibration

In order to solve the model numerically, it needs to be calibrated and we summarize the parameter values in Table 1.

Preferences and Production The household's discount factor β is set to 0.99, reflecting a real interest rate of 3.3%. The elasticity of labor supply η takes value 1 and χ is chosen such that value of leisure in the non-stochastic steady state equals to 30% of time endowment. The demand shock Z_t ranges from 0.99 to 1.1, so that the lowest optimal nominal rate is 0 and the highest around 10%. The supply shock A_t is assumed to fluctuate between $\pm 10\%$ of the mean.

Labor Markets The elasticity of substitution among labor varieties ϵ is set to equal 4.5, to be consistent with an average unemployment rate of 5% when labor is indivisible, in line with Galí(2011). Wage rigidity is captured by the convexity parameter ϕ and the asymmetry parameter ψ in the adjustment cost function (10). ϕ is set to be 32, which can be translated in a Calvo probability of not changing wages of 0.76 per quarter. We set ψ equal to 1,077,970(Kim and Ruge-Murcia 2009).

Table 1: Parameters

	value	Target
discount factor	$\beta = 0.992$	U.S. annual interest rate 3.3%
elasticity of labor supply	$\eta = 1$	
	$\chi = 2$	30% leisure time
supply shock	$A_t \in [0.9, 1.1]$	$\pm 10\%$ from 1
demand shock	$Z_t \in [0.99, 1.1]$	$i^{FI} \in [0, 10\%]$
elasticity of substitution of labor	$\epsilon = 4.5$	natural unemployment rate 5%
Convexity in wage adj. cost function	$\phi = 32$	Calvo probability 0.76
Asymmetry in wage adj. cost function	$\psi = 1,077,970$	Kim and Ruge-Murcia (2009)

6 Results

We now show the computational solution of the optimal policy under partial information in the model of Section 4.

We plot the optimal policy under full information and partial information in the same figure for comparison purpose. In Figure 4 the yellow region is the set of all equilibrium pairs (Π_t^p, i_t) that could have been realized under full information. The central bank adopt strict wage inflation target under full information, so $\Pi_t^w = 1$ always holds. Since the price setting is flexible, the firm always set $P_t = \frac{W_t}{A_t}$. As a result, Π_t^p ranges from $\frac{A_{t-1}}{A_{\max}}$ to $\frac{A_{t-1}}{A_{\min}}$, which means in equilibrium under full information, any price inflation(deflation) is purely caused by supply shock and demand shock plays no role in it. Then for any given level of price inflation, any level of demand shock could be realized, accompanied with the central bank's policy rate $i_t^{FI} = \frac{Z_t}{\beta} - 1$ to fully stabilize it. That is why the set of all equilibrium pairs (Π_t^p, i_t) consists of a rectangle.

As for the case of partial information, the red line plots the policy rate i_t against Π_t^p according to $i_t = \mathcal{R}^*(\Pi_t^p)$, computed using Algorithm 1. The intuition for the results is as follows.

First, we can find that the policy rule is an increasing function of price inflation Π_t^p, i.e. $\mathcal{R}'(\Pi^p) > 0$. We know that the central bank should fully stabilize demand shock if it has full information about the economy. But under the scenario of partial information, the central

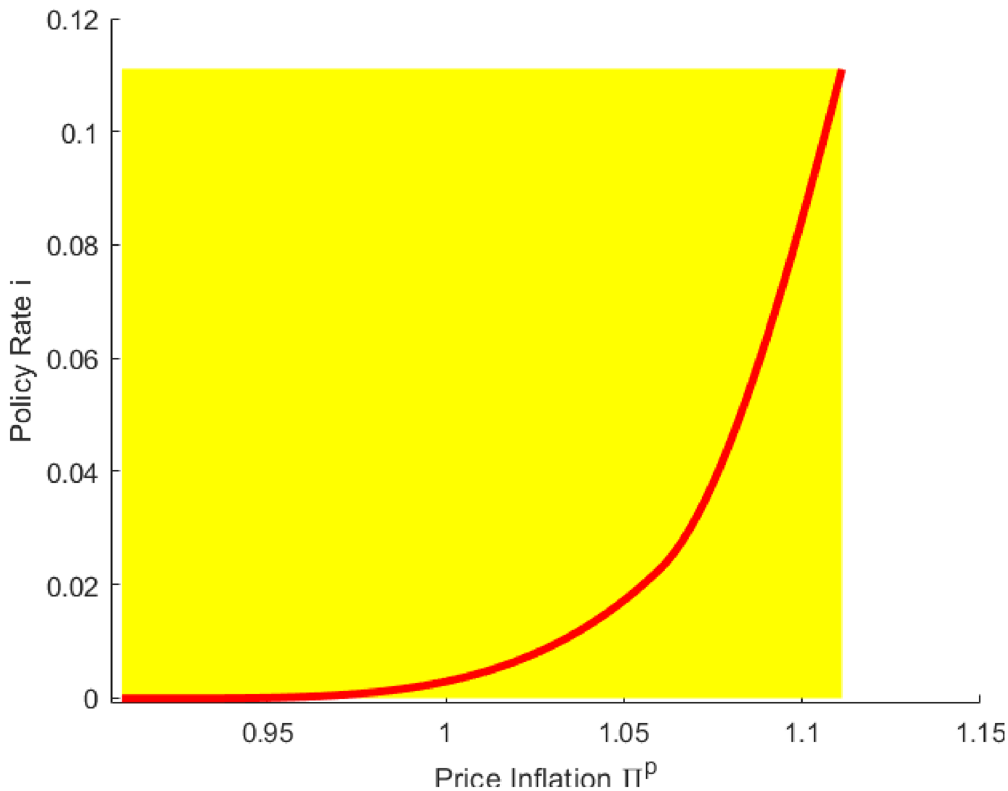
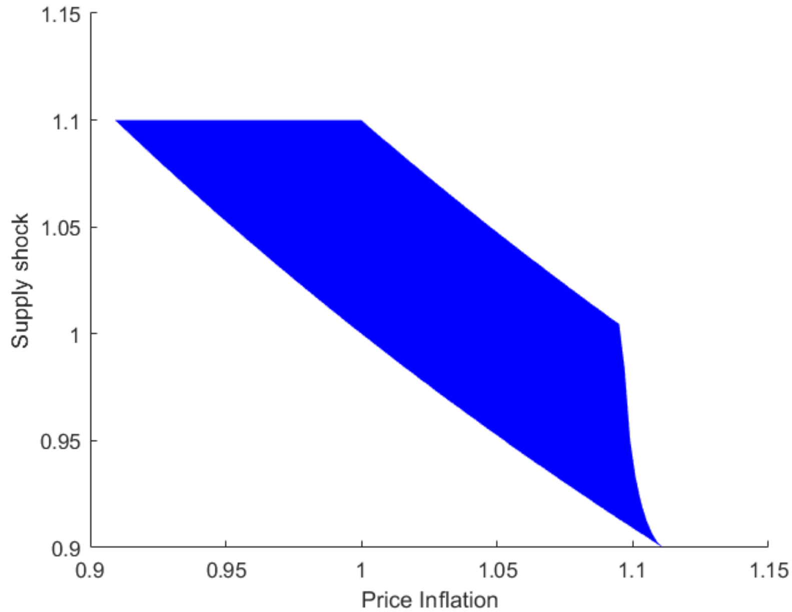
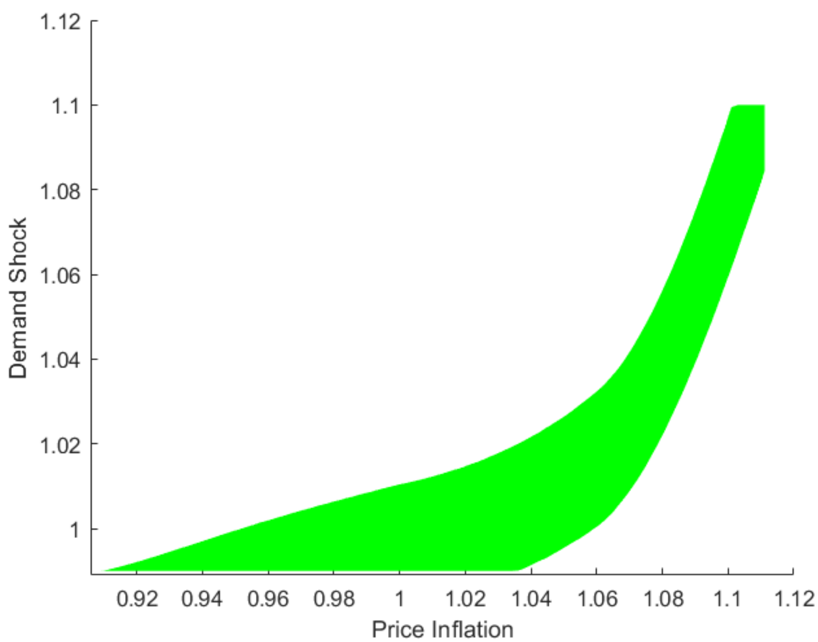


Figure 4: Optimal policy under FI and PI. Thick red line: $\mathcal{R}^*$; yellow region: set of FI pairs (Π_t^p, i_t) for all possible realizations of (A_t, Z_t); black dashed line: zero policy rate



(a) Set of admissible supply shocks $\mathbf{A}(\Pi_t^p, \mathcal{R}^*(\Pi_t^p)))$



(b) Set of admissible demand shocks $\mathbf{Z}(\Pi_t^p, \mathcal{R}^*(\Pi_t^p)))$

Figure 5: Set of admissible fundamental shocks consistent with $\mathcal{R}^*$

bank can only infer it from the signal, Π_t^p. Therefore, the responding rule of nominal rate to price inflation is hinged on how the signal Pi_t^p reveals about the demand shock Z_t. From the reaction function defined from Equation (27), one can get $h_Z > 0$ by applying implicit function theorem. The mechanism behind this can be explained by the Euler equation (11) and the wage Phillips curve (15): higher demand shocks will boost aggregate consumption and move the wage inflation upwards along the wage Phillips curve hence increase price inflation. To stabilize the demand shock, $\mathcal{R}^*$ should also be an increasing function of Π_t^p.

Apart from being increasing, we see $\mathcal{R}^*$ is non-linear: the higher the price inflation is, the more forcefully the central bank will respond to it. We argue that how strongly the central bank should respond to the price inflation is decided by two factors: the informativeness of the signal and the effectiveness of the policy. The informativeness means how accurate the price inflation signals the demand shock. The more confidence the central bank has on the accuracy of the signal, the more determined it should respond to price inflation. The effectiveness means how easily the changes of policy rate can affect the economy. The more effective the policy rate is, the more cautious the central bank should move.

Figure (5) depicts the informativeness of the signal by showing the possible values of shocks that are compatible with each given level of price information Π_t^p and the policy rule $\mathcal{R}^*$. One can find that the set of the possible shocks that are compatible with a given signal, $\mathbf{A}(\Pi_t^p, \mathcal{R}^*(\Pi_t^p)))$ and $\mathbf{Z}(\Pi_t^p, \mathcal{R}^*(\Pi_t^p)))$ are narrowed down when Π_t^p moves towards its two extreme values. For the lowest and highest observation of price inflation, there is full revelation. The minimum value of Π_t^p is only consistent with the lowest possible Z_t and highest possible A_t. Given $\mathcal{R}^*(\Pi_t^p = \frac{1}{A_{\max}}) = \frac{Z_{\min}}{\beta} - 1$, any demand shock $Z_t > Z_{\min}$ will lead to $\Pi^w > 1$ hence $\Pi_t^p > \frac{1}{A_{\max}}$, which is inconsistent with the signal. But when $\Pi_t^p = \frac{\Pi_t^w}{A_t}$ increases, the central bank is uncertain about the true cause of the price inflation, an underact to the demand shock resulting higher Π_t^w or lower supply shock A_t. That is why we have more fundamental shocks that are compatible with the policy and the signal in the intermediate region. As Π_t^p gets high enough, the central bank becomes confident that wage inflation is happening and increase the policy rate sharply. When Π_t^p arrives its maximum $\frac{1}{A_{\min}}$, the central bank also raises the interest rate to the highest level $\frac{Z_{\max}}{\beta} - 1$. Conditional on observing $\Pi^p_{\max}$, $Z_{\max}$ is the only possible realization of demand shock. Any $Z_t < Z_{\max}$ will lead us to observe a lower price inflation Π_t^p.

Since the monetary policy aims to stabilize the demand shock, one can also interpret the monetary policy as a "mixed stategy" to the possible demand shocks. If we plot the optimal policy and the endogenous information set of demand shock, we can find that the nominal rate is more or less a weighted average of the possible demand shocks, as is shown in Figure 6.

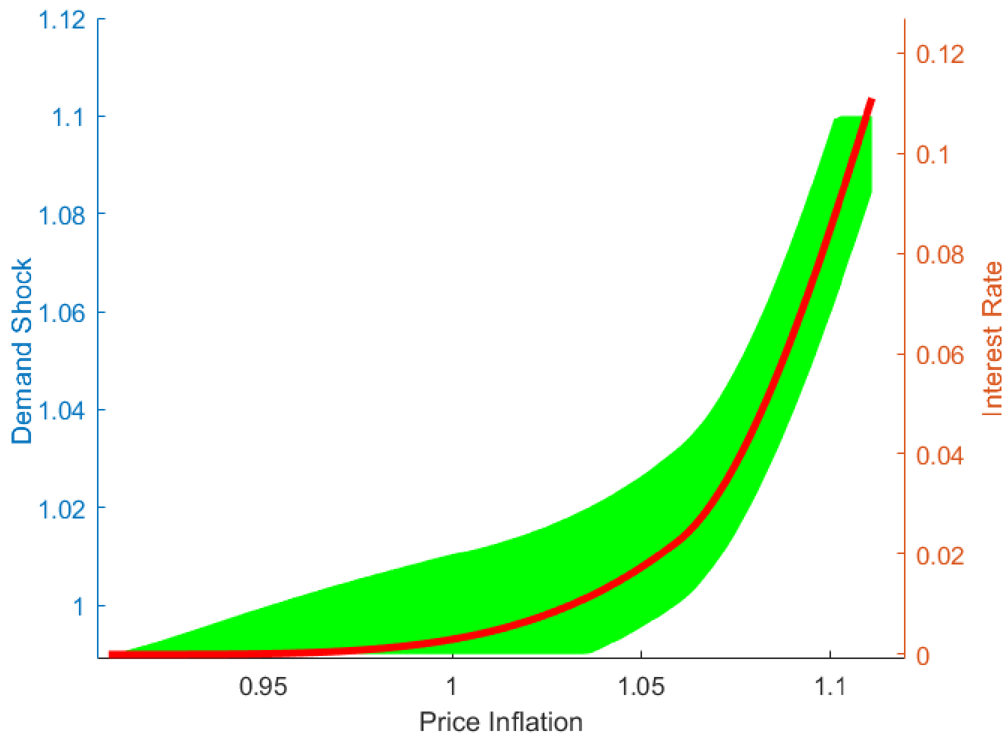


Figure 6: Optimal policy: a weighted average of possible demand shocks. The left y-axis (blue) is the scale for demand shocks and the right y-axis (red) is the scale for nominal rates

The other factor affecting the slope of the policy function is the effectiveness. The real effects of monetary policy is changing along the Phillips curve shown in Figure (3). When the wage inflation is low, the wage Phillips curve is flatten, the monetary policy is less transmitted as wage inflation and has a larger real effect on output. When the wage inflation is high, the wage Phillips curve is steep, the monetary policy is less effective and more transmitted as wage inflation. The real effect of monetary policy is determined by nominal wage rigidity. The higher the rigidity is, the more effective the monetary policy is. Downward nominal wage rigidity bends the Phillips curve and makes the monetary policy diminishing as wage inflation increases. So accordingly the central bank raises interest rate slowly when the monetary policy is very powerful but raises it more quickly when its real effect on aggregate demand is weak.

We can have a better understanding of the policy rule if we take both factors into account. When observing low level of price inflation, the central bank could extract a good signal about the demand shock Z_t, but still raise the policy rate slowly because the monetary policy has large real effects. In the intermediate region of Π_t^p, the monetary policy is less powerful but the central bank chooses to respond to price inflation mildly because it now faces much more uncertainty. When Π_t^p is high enough, the corresponding monetary policy rises sharply as there is less uncertainty now and the monetary policy is not so effective as it is in the low inflation regime.

7 Policy Comparison

In this section, we compare the optimal policy under partial information with some alternative policy rules, a simple Taylor rule, "certainty equivalence" and a "standard recipe" which can be seen as the expectation of the optimal policy rate under the probability measure of exogenous shocks. We plot these policies in Figure 7.

A simple Taylor rule $i_t = (\Pi_t^p)^\omega - 1$ where $\omega = 1.5$ captures how strongly the central bank responds to the inflation. Compared with optimal policy, the Taylor rule overreacts to price inflation and will lead to greater welfare loss as inflation gets higher.

The "standard" recipe,plotted in green line, can be seen as the expectation of the optimal policy rate under the measure of exogenous shocks

$$\int (\mathcal{U}_i^* + \mathcal{U}_\Pi^* h_i^*) f_Z(z) f_A(a) \mathrm{d}z \mathrm{d}a = 0 \tag{32}$$

One can find that its deviation from optimal policy becomes larger as inflation rate moves to the two extremes, which is not surprising as the "standard" recipe fails to take into account the endogeneity of the signal.

The certainty equivalence means that the central bank still adopts the function form the optimal policy under full information but replace the realizations of fundamental shocks with its best estimate, $i^{CE} = \frac{\mathbb{E}[Z_t]}{\beta} - 1$. One can find that the "certainty equivalence" prescribes an almost linear policy rule, which will cause great distortions in the middle region.

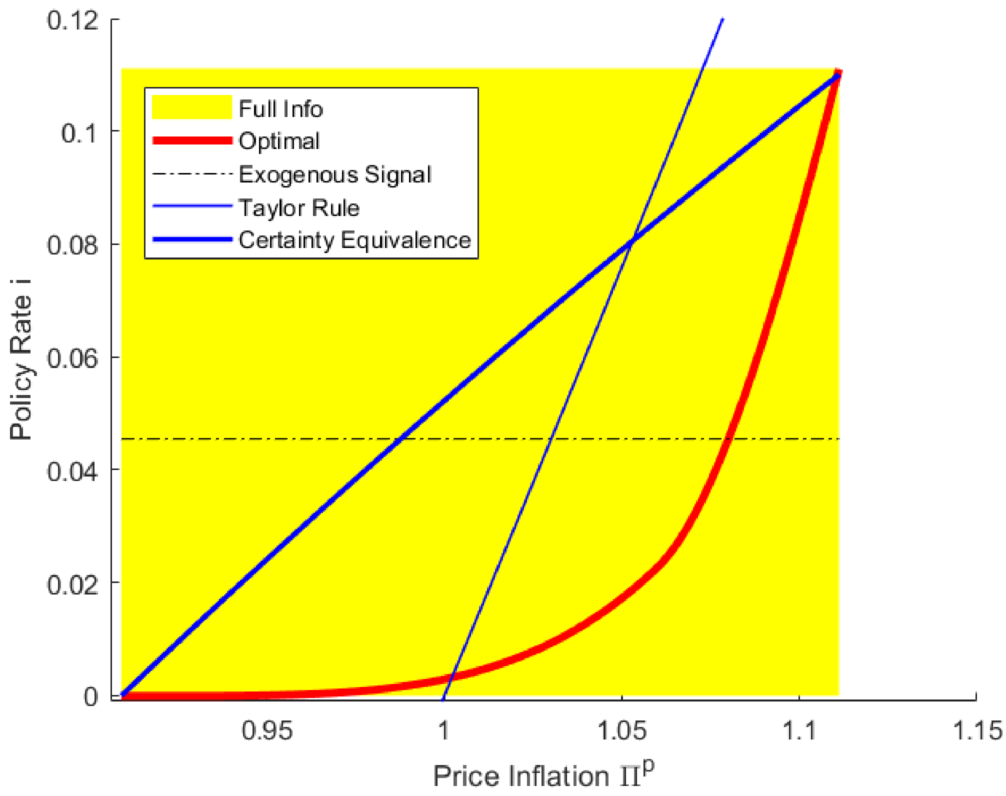


Figure 7: Policy comparison. Thick red line: $\mathcal{R}^*$; blue line: simple Taylor rule; dashed line: "Standard recipe", which treats signals to be exogenous; yellow region: set of FI pairs (Π_t^p, i_t) for all possible realizations of (A_t, Z_t); dotted line: zero policy rate

7.1 The endogeneity of the signal

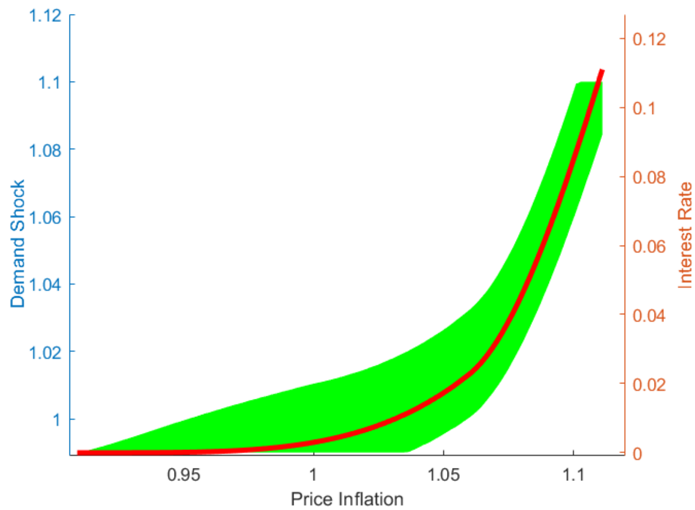
In this economy, the monetary policy aims for stabilizing demand shocks and can be seen as a weighted average of the latter. On the other hand, price inflation, the signal observed by the central bank is endogenous to the policy adopted, hence the set of possible of realizations of fundamental shocks is shaped by the monetary policy. In Figure 8, we plot the possible realizations of demand shocks that are consistent with the policy rules and the observations, represented by green color bars. At each given level of price inflation observed (x-axis), the height of the green bar represents the range of demand shocks could have been realized. The higher (wider) the bar is, the greater uncertainty the central bank faces. When the signal is taken as exogenous, policy choice has no effect on it and that is why we see a rectangle in the middle panel. As for the optimal policy and “certainty equivalence” principal, they have something in common: full revelation in extreme points, the signal of highest and lowest price inflation. But in the intermediate level of price inflation, the degree of uncertainty does not change much for the case of CE, in contrast to the case of optimal policy.

7.2 Welfare

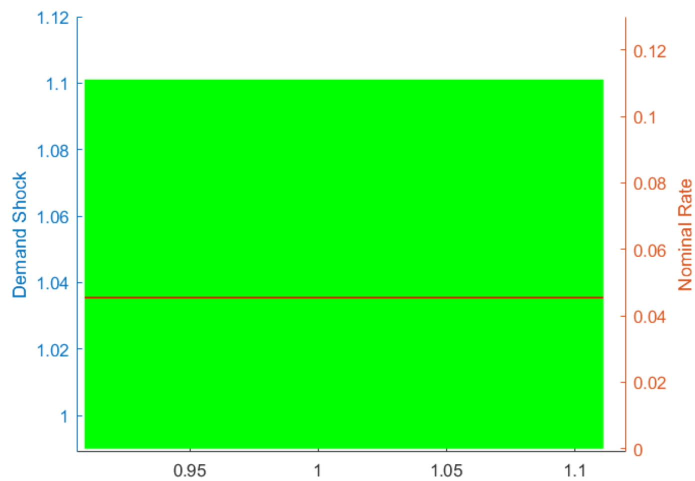
To further evaluate different policy rules, we now compute the unconditional mean of welfare for each policy rule (denoted as W) and their percentage differences with the welfare in an economy where the nominal rigidity is absent (denoted as W_{flex}), defined formerly as:

$$\Delta W = 100 \cdot [\exp(\mathbb{E}W_{flex} - \mathbb{E}W) - 1] \tag{33}$$

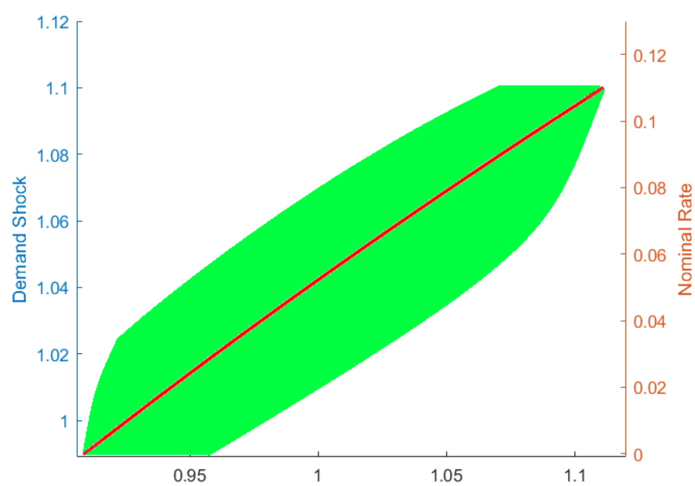
We can interpret this as a welfare loss from wage rigidity. The optimal policy under full information achieves its first best and therefore has 0 welfare loss. Table (2) presents the welfare loss for all the candidates policy rules under partial information. One can find that the optimal policy rule performs much better than the alternative choices.



(a) Set of admissible demand shocks $\mathbf{Z}(\Pi_t^p, \mathcal{R}^*(\Pi_t^p)))$



(b) Set of admissible demand shocks with “standard Recipe”



(c) Set of admissible demand shocks with “Certainty Equivalence”

Figure 8: Set of admissible demand shocks consistent with alternative policy rules

Table 2: Welfare loss

Full Information	Optimal	Certainty Equivalence	Taylor Rule	Standard Recipe
0	0.16	0.28	0.34	0.45

8 Serial correlated shocks

In our main example, the supply and demand shocks are independent identically distributed. The assumption of i.i.d. process for the exogenous variables is made for simplicity and illustrative purpose. To be consistent with empirical evidence, we solve the optimal monetary policy with serial correlated fundamental shocks.

8.1 Optimal policy under full information

To be more precise, we now assume the supply shock A_t and demand shock Z_t follow the AR(1) process with non-stochastic means normalized to unity:

$$\ln A_t = \rho_a \ln A_{t-1} + \varepsilon_{a,t} \tag{34}$$

$$\ln Z_t = \rho_z \ln Z_{t-1} + \varepsilon_{z,t} \tag{35}$$

The autoregressive parameters, ρ_a and ρ_z, lie between zero and one. The innovations, $\varepsilon_{a,t}$ and $\varepsilon_{z,t}$ are drawn from normal distributions of mean 0 and standard deviations σ_a and σ_z.

Under rational expectations, one can find the dynamic IS curve and the new Keynessian Phillips curve:

$$\frac{A_t^{1-\rho_a} Z_t^{1-\rho_z}}{C_t} = \beta(1+i_t)\mathbb{E}_t \frac{\exp(\varepsilon_{z,t+1})\exp(\varepsilon_{a,t+1})}{C_{t+1}\Pi_{t+1}^w} \tag{36}$$

$$\frac{\epsilon\chi Z_t^{1-\rho_z} C_t^2}{[A_t - \Phi(\Pi_t^w)]^2} + \frac{(1+\tau)(1-\epsilon)A_t Z_t^{1-\rho_z}}{A_t - \Phi(\Pi_t^w)} - \frac{Z_t^{1-\rho_z}\Phi'(\Pi_t^w)\Pi_t^w}{A_t - \Phi(\Pi_t^w)} + \beta\mathbb{E}_t \frac{\exp(\varepsilon_{z,t+1})\Phi'(\Pi_{t+1}^w)\Pi_{t+1}^w}{A_t^{\rho_a}\exp(\varepsilon_{a,t+1}) - \Phi(\Pi_{t+1}^w)} = 0 \tag{37}$$

The optimal policy under full information can be stated formerly as:

$$\max_{\{i_t, C_t, N_t, \Pi_t^w\}} U(C_t, N_t; Z_t) \tag{38}$$

$$\text{s.t. (14), (36), (37)}$$

As shown in the appendix, the optimal policy sets $\Pi_t^w = 1$ and $i_t = \frac{Z_t^{1-\rho_z}}{\beta} - 1$.

8.2 Optimal policy under partial information

Note that the expectation part of the Phillips curve is not a constant any more but depends on the supply shock A_t. We denote this part as $F(A_t)$. The expectation part of the IS curve is still a constant in equilibrium, which is denoted as G. Now one can find the function mapping from the policy variable i_t to the signal Π_t^p:

$$\Pi_t^p = g(i_t, A_t, Z_t) \tag{39}$$

as an implicit function from:

$$\frac{Z_t^{1-\rho_z}}{\beta(1+i_t)G} = \sqrt{\frac{A_t(A_t - \Phi(A_t\Pi_t^p))(\Phi'(A_t\Pi_t^p)\Pi_t^p - (1+\tau)(1-\epsilon))}{\epsilon\chi} - \frac{\beta F(A_t)(A_t - \Phi(A_t\Pi_t^p))^2}{\epsilon\chi Z_t^{1-\rho_z}}} \tag{40}$$

and the optimal policy problem under partial information can be stated as

$$\max_{\mathcal{R}:\mathcal{P}\to\mathcal{T}} \mathbb{E}[\mathcal{U}(i_t, \Pi_t^p, A_t, Z_t)] \tag{41}$$

$$\text{s.t. (25), (39)}$$

The optimal policy rule $\mathcal{R}^*$ satisfies the first-order condition:

$$\int_{-\infty}^{+\infty} \frac{\mathcal{U}_i^* + \mathcal{U}_\Pi^* g_i^*}{|g_Z^*|} f_Z(\mathcal{Z}^*(\Pi^p, a)) f_A(a) \mathrm{d}a = 0 \tag{42}$$

To solve out the optimal policy, we use a linear function to approximate the expectation part in (40), i.e. $F(A_t) \approx F_1 \ln A_t + F_2$ and then apply the following algorithm:

Algorithm 2 Given the optimal policy under full information, $i^{FI}(A, Z) = \frac{Z^{1-\rho_z}}{\beta} - 1$, one can find the pairs (Π^p, i) for all possible realizations of (A, Z). Since now we have unbounded shocks A_t and Z_t, the pairs (Π^p, i) fill the whole $\mathbb{R}^2$ space. Then follow the following steps:
(1) Discretize the set of possible values for Π^p. Here we choose to discretize the interval $\mathbf{A} = [\exp((1-\rho_a)a_{t-1} - 3\sigma_a), \exp((1-\rho_a)a_{t-1} + 3\sigma_a)]$;
(2) Guess an initial value for G and F_1, F_2, denoted as G^0 and F_1^0, F_2^0;
(3) Taking $G = G^0$ and $F_1 = F_1^0, F_2 = F_2^0$, for each value $\overline{\Pi^p}$ on the grid created in step 1, one can find the value i^* that solves the non-linear equation

$$\int_{\mathbf{A}} \frac{\mathcal{U}_i(i^*, \overline{\Pi^p}, a, z^*) + \mathcal{U}_\Pi(i^*, \overline{\Pi^p}, a, z^*) g_i(i^*, a, z^*)}{|g_Z(i^*, a, z^*)|} f_Z(z^*) f_A(a) \mathrm{d}a = 0 \tag{43}$$

where z^* denote the value of z solving the equation $\overline{\Pi^p} - g(i^*, a, z) = 0$ at a given a;
(4) Given the policy rule found in step 3, one can find the updated value for G and F_1, F_2, denoted as G' and F_1', F_2'; If $|G - G^0| < \varepsilon$ and $|F_1 - F_1^0| < \varepsilon, |F_2 - F_2^0| < \varepsilon$, where ε is the convergence criterion, stop; If not, take $G^0 \leftarrow G'$, $F_1^0 \leftarrow F_1', F_2^0 \leftarrow F_2'$ and go back to step (3).

8.3 Results

Now we show the computational solution of optimal policy under partial information when the shocks are serial correlated in Figure 9. The yellow region represents the set of all equilibrium pairs (Π_t^p, i_t) that could have been realized under full information as before. Recalling that all equilibrium pairs (Π_t^p, i_t) consist of a rectangle when shocks are uniformly distributed. But this is not the case when we have serial correlated shocks. Now the equilibrium pairs (Π_t^p, i_t) fill the whole $\mathbb{R}^2$ space. For convenience, we plot the area when $\varepsilon_{a,t}$ and $\varepsilon_{z,t}$ falls within 3 standard deviations of their mean 0.

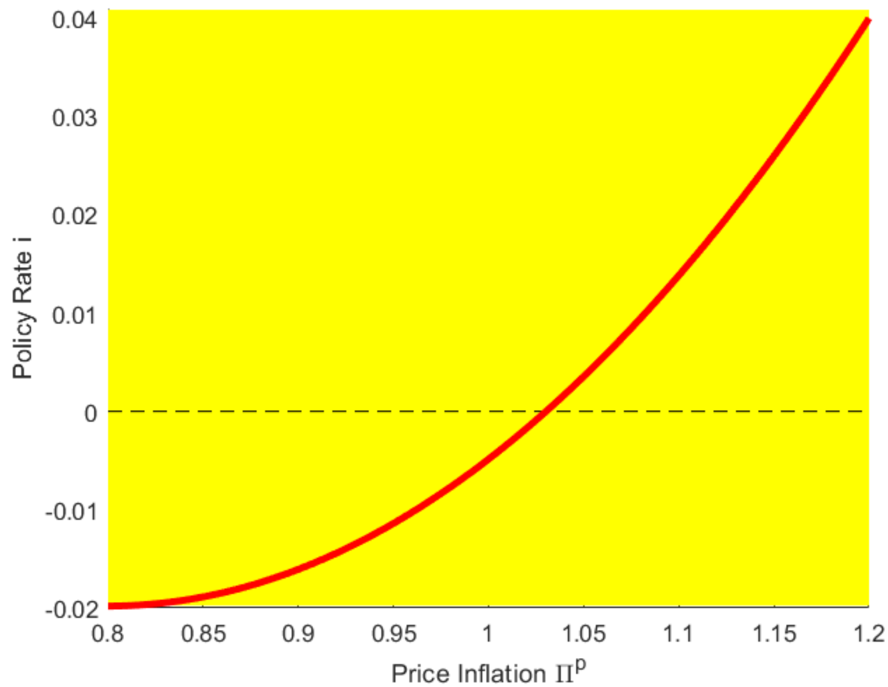


Figure 9: Optimal policy under FI and PI. Thick red line: $\mathcal{R}^*$; yellow region: set of FI pairs (Π_t^p, i_t) for all possible realizations of (A_t, Z_t); black dashed line: zero policy rate

One can find that the optimal policy rate is still an increasing function of price inflation as before. This is because higher price inflation signals higher demand shock unambiguously as the i.i.d. case and the monetary policy aims for stabilizing the demand shock. The non-linearity of the policy rule can also be attributed to the changing nominal rigidity and uncertainty along the Phillips curve and lead inertial behavior responding to low inflation and strong reaction to high inflation.

9 Conclusion

This paper explicitly analyzes the optimal monetary policy with signal extraction in a non-linear model with asymmetric wage adjustment cost. We find that the asymmetric wage adjustment cost and the signal endogeneity are the two forces shaping the responsiveness of policy rate to price inflation. The asymmetric wage adjustment cost changes the effectiveness of the monetary policy along the Phillips curve via changing the nominal rigidity. The signal endogeneity changes the central banks certainty level about the economy as the price inflation varies. These two factors make the monetary policy exhibits non-linear behavior.

References

Akerlof, G. A., Dickens, W. T., Perry, G. L., Gordon, R. J., & Mankiw, N. G. (1996). The macroeconomics of low inflation. Brookings papers on economic activity, 1996(1), 1-76.

Aoki, K. (2003). On the optimal monetary policy response to noisy indicators. Journal of monetary economics, 50(3), 501-523.

Daly, M. C., & Hobijn, B. (2014). Downward nominal wage rigidities bend the Phillips curve. Journal of Money, Credit and Banking, 46(S2), 51-93.

Fahr, S., & Smets, F. (2010). Downward wage rigidities and optimal monetary policy in a monetary union. Scandinavian Journal of Economics, 112(4), 812-840.

Hauk, E., Lanteri, A., & Marcet, A. (2021). Optimal policy with general signal extraction. Journal of Monetary Economics, 118, 54-86.

Kim, J., & Ruge-Murcia, F. J. (2009). How much inflation is necessary to grease the wheels?. Journal of Monetary Economics, 56(3), 365-377.

Svensson, L. E., & Woodford, M. (2003). Indicator variables for optimal policy. Journal of monetary economics, 50(3), 691-720.

Svensson, L. E., & Woodford, M. (2004). Indicator variables for optimal policy under asymmetric information. Journal of Economic Dynamics and Control, 28(4), 661-690.