Optimal fiscal policy with Ricardian and hand-to-mouth agents

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Abstract

We study the optimal fiscal policy in a model with two types of agents who are different in their access to the financial markets: Ricardian agents have full access to the financial markets while the hand-to-mouth agents are constrained and could only consume their labor income in each period. We find that the optimal labor-tax is more volatile compared with a representative-agent economy without physical capital and the volatility is captured by the equilibrium condition that these two types of agents are faced with the same proportional labor tax. When capital is introduced to this economy, we find that in the long run capital tax should still be zero in the deterministic case. But the ex ante capital tax in the stochastic economy is again disturbed by the same proportional labor tax condition, which makes it fluctuate around zero instead of staying there.

Keywords: fiscal policy, heterogeneous agents, hand-to-mouth

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JEL Codes:

1 Introduction

How should a government use the fiscal instruments when faced with shocks to the government expenditure? Ramsey optimal tax theory gives two important insights into this question: taxes on labor income should be smoothed and government should issue bonds to buffer the shocks (Barro [1979]; Lucas Jr and Stokey [1983]; Kingston [1991]; Zhu [1992]), while long- run capital tax should be set to zero (Chamley [1986]; Judd [1985]). These cornerstone results are all based on the assumption of a representative agent in the economy. Therefore they are all forward-looking and supposed to adjust their consumption and labor supply based on the tax and interest rates. However the strong response of aggregate consumption to interest rate changes that accounts for the large direct effects in representative agent models is questionable in light of empirical evidence. Macro-econometric analysis of aggregate time-series data finds a much smaller sensitivity of consumption to changes in the interest rate. The aggregate data should be viewed as generated by two types of agents: one forward-looking and consuming their permanent income; the other, behaving impatiently and spending its current income (Campbell and Mankiw [1989], Campbell and Mankiw [1991]).

If a significant fraction of agents are constrained in the financial markets, then they will only adjust their consumption to tax changes but not to the interest rate. Then what confidence can we have that tax recommendations obtained in a representative economy can minimize the total cost of distortion? Because equating taxes over time does not mean equating the marginal cost of distortions over time, which is not optimal any more. Then what is the optimal fiscal policy in an economy with forward-looking agents and hand-to-mouth agents? We try to answer this question in this chapter.

The model economy is inhabited by agents that differ in their access to the financial markets. Hand-to-mouth agents are constrained in the financial markets while Ricardian agents are not. Lump-sum tax is ruled out. In the first scenario, we study the optimal fiscal policy in an economy without capital described by Lucas Jr and Stokey [1983]. Government uses flat-rate labor income tax and state-contingent bond to finance its expenditures. We find that, when government is not allowed to levy discriminatory labor tax, the optimal tax rate is not constant any more, even if we adopt the utility function that is homogeneous of consumption and labor supply and generates perfect constant tax rate in the representative economy. We also find that the more social planer cares about the hand-to-mouth agents, the more positively the optimal tax rate responds to the government expenditure. Government uses taxes to manipulate the prices of government bond and necessarily affects the inter temporal budget constraints of the Ricardian agents. If government is sided with the Ricardian agents, they will borrow at a low interest rate and lend at a high rate and vice versa.

In the second scenario, when capital is introduce to the model, we have indeterminacy of capital income tax and bond issuing. Follow Zhu [1992], we study the ex-ante capital tax rate in this economy and find that the fluctuations of capital tax is again captured by the equality condition of labor-income tax rates across agents.

My paper is related to two main strands of the literature. On the one hand, the paper builds on the earlier literature on the optimal policy, including Lucas Jr and Stokey [1983], Zhu [1992] ,Chari et al. [1994]. The closest forebears to our framework is Bassetto [2014]. He studies how the relative political power of "taxpayers affect the fiscal policies of a country in peace time and war time. Werning [2007] focuses on the distributional effects of distortionary taxes. In his paper, the introduction of hand-to- mouth agents solves the indeterminacy problem arised by lump-sum tax. On the other hand, the literature that links high MPC with hand-to-mouth agents. Campbell and Mankiw [1989] provides empirical evidence of the hand-to-mouth agents. Kaplan and Violante [2014] show that uninsurable risk, combined with the co-existence of liquid and illiquid assets in financial portfolios leads to the presence of a sizable fraction of poor and wealthy hand-to-mouth households, as in the data. Cloyne et al. [2016] show that households with mortgage debt exhibit large and significant consumption responses to tax changes.Debortoli and Galí [2017] try to study the monetary transmission mechanism with a simple two-agent economy.

The rest of the chapter proceeds as follows. In Section 2, we study the optimal proportional labor tax in the complete market, which follows Lucas Jr and Stokey [1983] in an economy without capital; In Section 3, we solve the model numerically. In section 4, we study the optimal long run capital tax in a deterministic case (Chamley [1986]) and stochastic case (Zhu [1992]) respectively. Section 5 concludes.

2 The Model

We consider an economy with two types of households: The first type of households are hand-to-mouth. They have no access to the financial markets and consume their after-tax labor income every period, which are denoted by K. The second type of agent have full access to the financial markets, which are denoted by R. Both types of households have the same preferences, which are given by a utility function:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \tag{1}$$

where C_t is consumption and N_t is labor supply. We adopt the following utility function:

$$U(C,N) = u(C_t) + v(N_t) = \log(C_t) - D\frac{N_t^{\gamma+1}}{\gamma+1} \tag{2}$$

so that the optimal labor tax rate is perfect constant in the representative agent economy described by Lucas Jr and Stokey [1983]. It would be convenient for us to compare the results.

The fraction of constrained households and unconstrained households are λ and $1-\lambda$ respectively.The technology follows the same spirit of Lucas Jr and Stokey [1983]. Firms are operated in a perfect competitive market with linear production function of labor input $y_t = f(l_t) = l_t$. Let g_t denote government purchases at time t. Then the resources constraint is

$$g_t + \lambda C_t^k + (1-\lambda)C_t^R = \lambda N_t^K + (1-\lambda)N_t^R \tag{3}$$

The government could levy a proportional tax on the labor income τ_t^n and issue the government debt $b_t^g(g_{t+1})$ contingent on future spending. I also assume that the tax rate is constrained to be equal across both types of agents and the marginal tax rate is constant on all labor income. The government budget constraint is

$$g_t + b_{t-1}^g(g_t) = \lambda\tau_t^n N_t^K + (1-\lambda)\tau_t^n N_t^R + \sum_{g_{t+1}|g^t} p_t(g_{t+1})b_t(g_{t+1}) \tag{4}$$

The hand-to-mouth agents' budget constraint is:

$$C_t^K = (1-\tau_t^n)N_t^K \tag{5}$$

The Recardian agents could buy state-contingent government bonds, so their budget constraint is

$$C_t^R + p_t^g(g_{t+1})b_t^R(g_{t+1}) = b_{t-1}^R(g_t) + (1-\tau_t^N)N_t^K$$

Note that since the population of Ricardian agents is $1-\lambda$, the bonds held by them satisfies $(1-\lambda)b_t^R = b_t^g$.

2.1 Competitive Equilibrium and Ramsey outcome

The household first-order-condition require that the price of government bonds satisfies

$$p_t^g(g_{t+1}) = \beta \frac{u'(C_{t+1}^R(g^{t+1}))}{u'(C_t^R)} prob(g_{t+1}|g_t) \tag{6}$$

and that taxes satisfy

$$1-\tau_t^N = -\frac{v'(N_t^R)}{u'(C_t^R)} = -\frac{v'(N_t^K)}{u'(C_t^K)} \tag{7}$$

We use these expressions to eliminate the prices and taxes in the hand-to-mouth agents' budget constraints, i.e. $C_t^K = (1-\tau_t^N)N_t^K = -\frac{v'(N_t^K)}{u'(C_t^K)}N_t^K$

$$u'(C_t^K)C_t^K + v'(N_t^K)N_t^K = 0 \tag{8}$$

The special utility function (2) allows us to eliminate C_t^K and solve N_t^K explicitly from (8), which is invariant to tax rate change, $N_t^K = N^K = D^{-\frac{1}{\gamma+1}}$. They only adjust their consumption level to respond tax rate change. In other words, their marginal propensity to consume (MPC) equals 1. Since markets are complete, the Ricardian agents can choose their optimal contingent plans based on a single Arrow-Debreu budget constraint:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u'(C_t^R) C_t^R + v'(N_t^R) N_t^R] = b_{-1}^R(g_0) u'(C_0^R) \tag{9}$$

where $b_{-1}^R(g_0)$ is the amount of government bonds held by each Ricardian agent and the total quantity of government bond $b_{-1}^g(g_0) = (1-\lambda) b_{-1}^R(g_0)$.

DEFINITION 1 Given initial bond holdings $b_{-1}^R(g_0)$ by the Ricardian agents, a *competitive equilibrium* is a sequence of taxes τ_t^N, prices $\{p_t^g(g_{t+1}), w_t\}$, and non-negative quantities $\{c_t^K, N_t^K\}$, $\{c_t^R, N_t^R, b_t^R(g_{t+1})\}$ such that

(i) hand-to-mouth agents choose $\{c_t^K, N_t^K\}$ to maximize their expected utility (2) subject to the budget constraint (8), taking prices and taxes as given;

(ii)Ricardian agents choose $\{c_t^R, N_t^R, k_t\, b_t(g_{t+1})\}$ to maximize the same utility form (2), taking $\{p_t^g(g_{t+1}), w_t\}$ as given;

(iii) Firms maximize profits: the equilibrium wage $w_t = 1$;

(iv) the government budget constraint (4) holds;

(v) markets clear: the resource constraints (3) hold for all periods t and histories $\{g_t\}_{t=0}^{\infty}$.

The Lagrangian for the Ramsey problem can be represented as:

$$\begin{aligned}
\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \{ & \alpha[u(C_t^K) + v(N_t^K)] + (1-\alpha)[u(C_t^R) + v(N_t^R)] \\
& + \nu_t [u'(C_t^K) C_t^K + v'(N_t^K) N_t^K] \\
& + \mu_t [u'(C_t^K) v'(N_t^R) - u'(C_t^R) v'(N_t^K)] \\
& + \theta_t [g_t + (1-\lambda) C_t^R + \lambda C_t^K - (1-\lambda) N_t^R - \lambda N_t^K] \\
& + \phi [u'(C_t^R) C_t^R + v(N_t^R) N_t^R - b_{-1}(g_0) u'(C_0^R)] \}
\end{aligned}$$

The government budget constraint is not explicitly included because it is redundant when the agents' budget constraints are satisfied and the resources constraint holds. To avoid the time inconsistency problem and make model easier, I assume that the outstanding government debt in the initial period b_{-1} is 0. So the first order conditions for the Ramsey problem are:

$$[C_t^K] : \alpha u'(C_t^K) + \nu_t[u''(C_t^K)C_t^K + u'(C_t^K)] + \mu_t v'(N_t^R)u''(C_t^K) + \lambda\theta_t = 0 \tag{10}$$

$$[N_t^K] : \alpha v'(N_t^K) + \nu_t[v''(N_t^K)N_t^K + v'(N_t^K)] - \mu_t u'(C_t^R)v''(N_t^K) - \lambda\theta_t = 0 \tag{11}$$

$$[C_t^R] : (1-\alpha)u'(C_t^R) + \phi[u''(C_t^R)C_t^R + u'(C_t^R)] - \mu_t v'(N_t^K)u''(C_t^R) + (1-\lambda)\theta_t = 0 \tag{12}$$

$$[N_t^R] : (1-\alpha)v'(N_t^R) + \phi[v''(N_t^R)N_t^R + v'(N_t^R)] + \mu_t u'(C_t^K)v''(N_t^R) - (1-\lambda)\theta_t = 0 \tag{13}$$

We can solve the competitive allocation $C_t^i, N_t^i, i = K, R$ as a function of g_t and ϕ from these four first order conditions and equations (3), (8) and (9) . That means, if government purchases are equal after two histories g^t and $g^{\tilde{t}}$ for $t, \tilde{t} > 0$, i.e.,

$$g_{t+1} = g_{\tilde{t}+1}$$

then the Ramsey choices of consumption and leisure, $\{C_{t+1}^i, N_{t+1}^i\}$ and $\{C_{\tilde{t}+1}^i, N_{\tilde{t}+1}^i\}$, are identical, which asserts that the optimal allocation is a function of the currently realized government purchases g_t only and does not depend on the specific history preceding realizations of g^t. Combining Ricardian agents' F.O.C.s (12) and (13):

$$\begin{aligned} (1-\alpha)[u'(C_t^R) + v'(N_t^R)] + \phi[u''(C_t^R)C_t^R + u'(C_t^R) + v''(N_t^R)N_t^R + v'(N_t^R)] \\ + \mu_t[u'(C_t^K)v''(N_t^R) - v'(N_t^K)u''(C_t^R)] = 0 \end{aligned} \tag{14}$$

and the equilibrium conditions of labor market $1 - \tau_t^N = -\frac{v'(N_t^i)}{u'(C_t^i)}$ for $i = \{N, K\}$. One can find a more intuitive expression for the optimal tax rate. Assume first $\mu_t = 0$, so that government can levy agent specific tax, then the optimal taxation is similar to the results in Lucas and Stokey economy:

$$\tau_t^K = \frac{\nu_t(1+\gamma)}{\alpha}$$

$$\tau_t^R = \frac{\phi(1+\gamma)}{1-\alpha}$$

i.e. the labor tax for Ricardian agents would still be a constant and the government use tax only to adjust the hand-to-mouth agents' consumption and labor supply. But equation (7) imposes equality of labor-income tax rates across agents, so the optimal tax rate is not constant any more and its volatility is captured by the second line of equation (14), where μ_t is the Lagrange multiplier associated with the equality constraint of labor income tax rate across 2 agents. Another way to analyze the problem is to find out the competitive allocation associated with a perfect constant tax rate. In this case, the hand-to-mouth agents achieves perfect consumption and leisure smoothing. Considering the resources constraint (3) under this assumption:

$$g_t + \lambda C^K + (1-\lambda)C_t^R = \lambda N^K + (1-\lambda)N_t^R$$

All the shocks of government expenditures would be born by the Ricardian agents, which is not optimal from the perspective of a benevolent government.

3 Quantitative Analysis

3.1 Calibration

To provide a quantitative illustration of the role of heterogeneity, we consider a calibration of the model where the share of hand-to-mouth agents is set to $\lambda = 0.5$, following Campbell and Mankiw [1989]. The parameter D is calibrated so that in the non-stochastic steady state with government debt and deficit equal to zero, the labor supply is 70 per cent of the time endowment.We assume the government spending follows an AR(1) process:

$$g_t = (1 - \rho)\bar{g} + \rho g_{t-1} + \epsilon_t$$

The rest of the parameters are calibrated as following:

Table 1: Parameters of 2-agents Model

Parameters	Values
share of Keynessian agents λ	0.5
discount rate β	0.99
D	2
γ	1
time endowment	1
$\bar{g}$	0.175
ρ	0.95
variance of shock $\sigma^2(\epsilon)$	0.012^2

3.2 Fiscal policy of a benevolent government

First, we consider the fiscal policy of a benevolent government, which sets the Pareto weight of different agents equal to their population share. Figure 1 shows the simulated paths of government expenditures, competitive allocations and the tax rate, in contrast to two alternative extreme policy: one is to balance the budget period by period without issuing any bonds, the other is to impose a perfect constant tax rate, as the government does in a representative economy. When the government balances its budget constraints period by

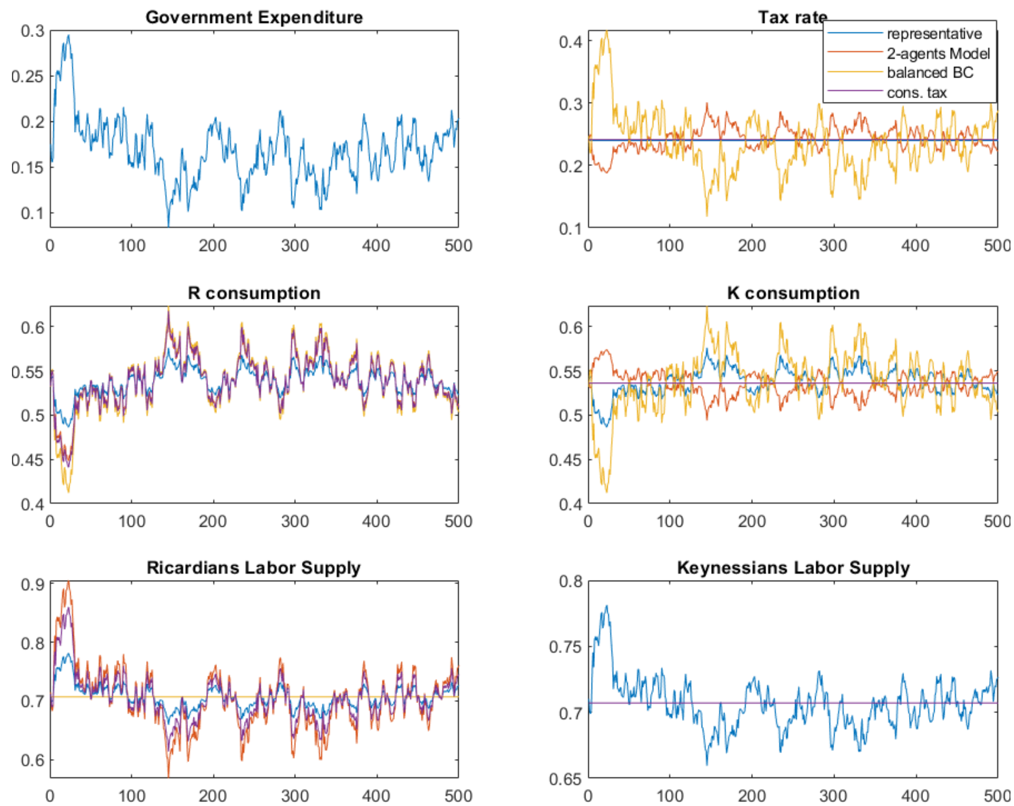

Figure 1: Competitive allocations under alternative policies

period, it cannot issue public debt to buffer the expenditure shock. As a result, all the agents in the economy would be hand-to-mouth and they work for a fixed amount of time every period. In such an economy, the government expenditures perfectly correlate with tax rates and consumption, positive and negative respectively. We would observe the most volatile consumption in this no-bond world.

Now let's evaluate the constant tax rate policy. Since only a fraction of the population could hold the public debt which helps to buffer the expenditure shock, the government has to levy a slightly higher tax rate (0.2415-0.2398=0.0017) to achieve perfect insurance. Why is it not optimal? Because the government could use tax rate to change states prices and distribute the distortions more evenly across time. When the expenditure is high, government lowers labor tax rate and encourage the Ricardian agents to work more and lowers state price. That's why we could observe more volatile labor supply of the Ricardian agents when government adopt the optimal policy.

To further illustrate the welfare implications of different policies, we can find the consumption-equivalent welfare gains compared with an economy where the government cannot issue debt but only finance its expenditures with taxes. An interesting fact is that, when a constant tax rate is imposed, hand-to-mouth agents get fully insured at the cost of Ricardian agents.

Table 2: Welfare gains in terms of CE

	No debt	Representative	Benevolent	Constant Tax
Hand-to-mouth	0	0.1733	0.0279	0.8165
Ricardian	0	0.1733	0.1726	-0.4542

3.3 Debt or tax? a redistribution concern

Government will choose different strategies to buffer the expenditure shock when it favors different agents. Since only the Ricardian agents hold public debt, the government could affect their welfare by distorting the state prices when they save or disave. We can define the Ricardian agents' net savings as

$$S_t = (1 - \tau_t)N_t^R + b_{t-1}(g_t) - C_t^R$$

If the government sides with the Ricardian agents, it increases the state prices when Ricardian agents save ($S_t > 0$) and lowers state prices when $S_t < 0$, i.e. the Ricardian agents sell high and buy cheap. In Figure 3, I plot the reaction functions when the government put different weights on the agents. α is the Pareto weight on the hand-to-mouth agents. Lower α (yellow line) corresponds to the policy beneficial to the Ricardian agents, who save when the government expenditure g_t is low and vice versa. One can find that the consumption of Ricardian agents is relatively low when they save, which means higher state price determined by $u'(C_t^R)$ of their savings. The implication of fiscal policy here is that the government employs taxes to distort inter-temporal prices to affect agents' wealth.

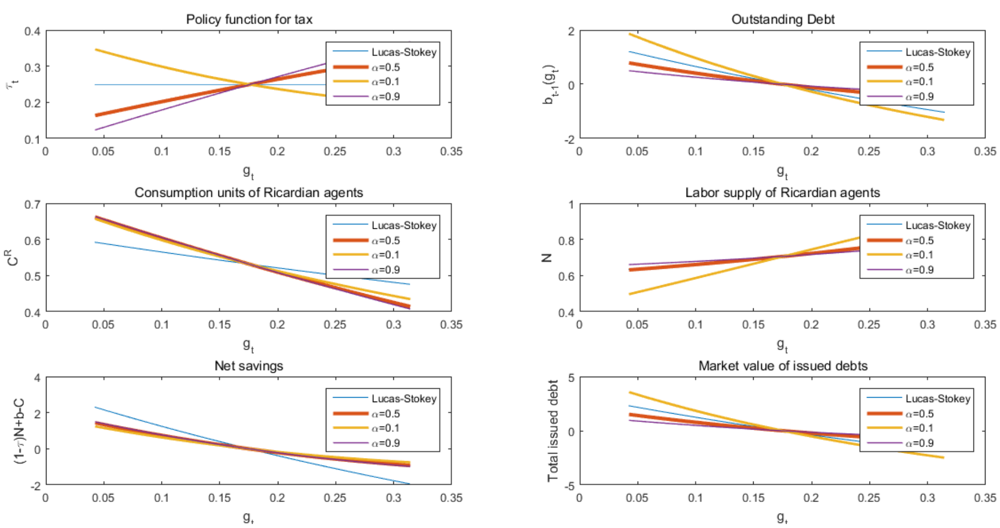

Figure 2: Reaction functions with different Pareto weights

One can also find the Pareto frontier when the government puts different Pareto weights on each group of agents.

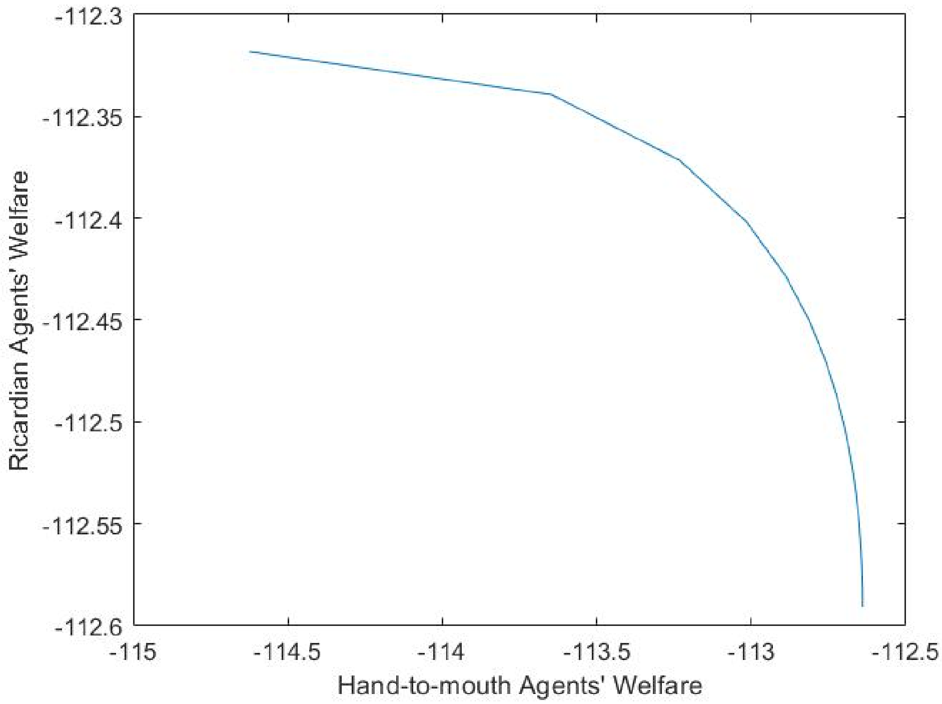

Figure 3: Pareto Frontier

4 Extensions to an economy with capital

This section extends the analyses of Ramsey taxation to an economy with capital accumulation. I use a stochastic version of a one-sector neoclassical growth model in discrete time and infinite horizon. The households' preferences are ordered by:

$$\sum_{t=0}^{\infty}\sum_{g^t}\beta^t\pi_t(g^t)(u(C_t)+v(N_t)) \tag{15}$$

We follow the same spirit before: the hand-to-mouth agents have no access to the financial markets and could only consume their after-tax labor income in each period. The Ricardian agents could either buy government bonds or invest in the capital market.

4.1 Endowment and Technology

The Ricardian agents bring the initial capital k_{-1} to this economy and they supply labor together with the hand-to-mouth agents to the production firm. There is only one final good which can be either consumed or invested. The production function is constant to scale:

$$y_t = F(K_{t-1}, N_t)$$

There is a government in this economy and the government expenditure in units of consumption good in period t is denoted by g_t, which is assumed to be an exogenous stochastic process and the only source of uncertainty. The technology constraint follows:

$$\lambda C_t^K + (1-\lambda)C_t^R + g_t + (1-\lambda)(k_t - (1-\delta)k_{t-1}) = F((1-\lambda)k_{t-1}, \lambda N_t^K + (1-\lambda)N_t^R) \tag{16}$$

where δ is the depreciation rate of capital.

There are three perfectly competitive markets in the economy: the labor markets, the capital market, and the government bond market. The firm rents capital from consumers and the government trades one-period state-contingent claims with consumers. Given the government expenditure $\{g_t\}_{t=0}^{\infty}$, the government finances its exogenous purchase and debt obligation by levying flat-rate taxes on earnings from capital labor, at rates τ_t^K and τ_t^N respectively, and by issuing state-contingent bonds. I also assume that the tax rate in labor income is constrained to be equal across agents. Then the government budget constraint follows:

$$g_t + b_{t-1}^g(g_t) = \tau_t^N w_t(\lambda N_t^K + (1-\lambda)N_t^R) + \tau_{t-1}^K r_t(1-\lambda)k_{t-1} + \sum_{g_{t+1}|g^t} p_t^g(g_{t+1})b_t^g(g_{t+1}) \quad (17)$$

The timing of trading is a crucial issue in this economy. In period $t = 0$, the supply of capital is inelastic and the tax on the capital income is therefore not distortionary. So the government wants to tax the capital income in the initial period as heavily as possible to minimize distortion caused by other distortionary taxes. If it happens that the revenue collected from this tax is big enough to finance all the current and future government expenditures, then there is no need to use distortionary taxes. To make the exercise interesting we impose an upper bound on the period 0 capital tax so that the government does need to tax labor and capital income in the future periods. To avoid policy indeterminacy, capital taxes are not state-contingent but decided one period in advance. Only Arrow securities are used to complete the markets.

The hand-to-mouth agents do not have access to capital markets and could only consume their labor income, so their budget constraints remain unchanged:

$$C_t^K = (1 - \tau_t^N) w_t N_t^K \tag{18}$$

However the Ricardian agents' sequential budget constraints follows:

$$C_t^R + k_t + \sum_{g_{t+1}|g^t} p_t^g(g_{t+1}) b_t^R(g_{t+1}) = (1 - \tau_{t-1}^K) r_t k_{t-1} + (1 - \tau_t^N) w_t N_t^R + (1 - \delta) k_{t-1} + b_{t-1}^R(g_t) \tag{19}$$

where $(1 - \lambda) b_t^R(g_{t+1}) = b_t^g(g_{t+1})$ and $(1 - \lambda) k_t = K_t$, i.e. they have equal share to the government bonds and the capital.

4.2 Competitive equilibrium

Firms Since the factors markets are perfectly competitive, the firm's F.O.C implies:

$$r_t = F_K((1 - \lambda) k_{t-1}, \lambda N_t^K + (1 - \lambda) N_t^R) \tag{20}$$

$$w_t = F_N((1 - \lambda) k_{t-1}, \lambda N_t^K + (1 - \lambda) N_t^R) \tag{21}$$

Households The households problem is to maximize their expected utility function under the budget constraints, the solutions are characterized by the following first order conditions:

$$(1 - \tau_t^N) w_t = -\frac{v'(N_t^R)}{u'(C_t^R)} = -\frac{v'(N_t^K)}{u'(C_t^K)} \tag{22}$$

$$p_t^g(g_{t+1}) = \beta \frac{u'(C_{t+1}^R(g^{t+1}))}{u'(C_t^R)} prob(g_{t+1}|g^t) \tag{23}$$

$$u'(C_t^R) = \beta \mathbb{E}_t u'(C_{t+1}^R(g^{t+1}))[1 + (1 - \tau_t^K) r_{t+1} - \delta] \tag{24}$$

Under complete market condition, the Ricardian agents budget constraints could be summed into a single one:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t [u'(C_t^R) C_t^R + v'(N_t^R) N_t^R] = u'(C_0^R)[(1 + r_0 - \delta) k_{-1} + b_{-1}^R(g_0)] \tag{25}$$

where $\tilde{r}$ is the after-tax interest rate. The non-arbitrage condition implies

$$1 = \sum_{g_{t+1}|g^t} p_t^g(g_{t+1})[1 + (1 - \tau_t^K) r_{t+1}(g^{t+1}) - \delta] \tag{26}$$

DEFINITION 3.3 Given initial capital and bond holdings $\{K_{-1}, b_{-1}(g_0)\}$, a *competitive equilibrium* is a sequence of taxes $\{\tau_t^K, \tau_t^N\}$, prices $\{p_t^g(g_{t+1}), r_t, w_t\}$, and non-negative quantities $\{c_t^K, N_t^K\}$, $\{c_t^R, N_t^R, k_t\}$ such that

(i) Hand-to-mouth agents choose $\{c_t^K, N_t^K\}$ to maximize their expected utility (15) subject to the budget constraint (18) taking prices and taxes that satisfy (21) as given;

(ii) Ricardian agents choose $\{c_t^R, N_t^R, k_t, b(g_{t+1})\}$ to maximize their utility, taking $\{p_t^g(g_{t+1}), r_t, w_t\}$ as given;

(iii) Firms maximize profits: the first-order conditions (20) and (21) hold;

(iv) Government budget constraint (17) holds;

(v) Markets clear: the resource constraints (16) hold for all periods t and histories $\{g_t\}_{t=0}^{\infty}$

5 Analytical results

To simplify the problem, we further assume $b_{-1} = 0$ and $\tau^K_{-1} = 0$. The Lagrangian for the Ramsey problem is:

$$\begin{aligned}
\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \{ & \alpha[u(C^K_t) + v(N^K_t)] + (1-\alpha)[u(C^R_t) + v(N^R_t)] \\
& + \nu_t[u'(C^K_t)C^K_t + v'(N^K_t)N^K_t] \\
& + \mu_t[u'(C^K_t)v'(N^R_t) - u'(C^R_t)v'(N^K_t)] \\
& + \theta_t[g_t + \lambda C^K_t + (1-\lambda)C^R_t + (1-\lambda)(k_t - (1-\delta)k_{t-1}) \\
& - F((1-\lambda)k_{t-1}, \lambda N^K_t + (1-\lambda)N^R_t)] \\
& + \phi[u'(C^R_t)C^R_t + v'(N^R_t)N^R_t]\} - \phi k_{-1}(F_{K,0} + 1 - \delta)u'(C^R_0)
\end{aligned}$$

The first order conditions for $t > 0$:

$$[C^K_t] : \alpha u'(C^K_t) + \nu_t[u''(C^K_t)C^K_t + u'(C^K_t)] + \mu_t v'(N^R_t)u''(C^K_t) + \lambda\theta_t = 0 \tag{27}$$

$$[N^K_t] : \alpha v'(N^K_t) + \nu_t[v''(N^K_t)N^K_t + v'(N^K_t)] - \mu_t u'(C^R_t)v''(N^K_t) - \lambda\theta_t F_{N,t} = 0 \tag{28}$$

$$[C^R_t] : (1-\alpha)u'(C^R_t) + \phi[u''(C^R_t)C^R_t + u'(C^R_t)] - \mu_t v'(N^K_t)u''(C^R_t) + (1-\lambda)\theta_t = 0 \tag{29}$$

$$[N^R_t] : (1-\alpha)v'(N^R_t) + \phi[v''(N^R_t)N^R_t + v'(N^R_t)] + \mu_t u'(C^K_t)v''(N^R_t) - (1-\lambda)\theta_t F_{N,t} = 0 \tag{30}$$

$$[k_t] : \theta_t - \beta\mathbb{E}_t\theta_{t+1}(1 + F_{K,t} - \delta) = 0 \tag{31}$$

and $t = 0$:

$$[C^K_0] : \alpha u'(C^K_0) + \nu_0[u''(C^K_0)C^K_0 + u'(C^K_0)] + \mu_0 v'(N^R_0)u''(C^K_0) + \lambda\theta_0 = 0 \tag{32}$$

$$[N_0^K] : \alpha v'(N_0^K) + \nu_0[v''(N_0^K)N_0^K + v'(N_0^K)] - \mu_0 u'(C_0^R)v''(N_0^K) - \lambda\theta_0 F_{N,0} = 0 \tag{33}$$

$$\begin{aligned}[C_0^R] :&(1-\alpha)u'(C_0^R) + \phi[u''(C_t^R)C_t^R + u'(C_t^R)] - \mu_0 v'(N_0^K)u''(C_0^R) + (1-\lambda)\theta_0 \\ &- \phi k_{-1}(F_{K,0} + 1 - \delta)u''(C_0^R) = 0\end{aligned} \tag{34}$$

$$\begin{aligned}[N_0^R] :&(1-\alpha)v'(N_0^R) + \phi[v''(N_0^R)N_0^R + v'(N_0^R)] + \mu_0 u'(C_0^K)v''(N_0^R) - (1-\lambda)\theta_0 F_{N,t} \\ &- \phi k_{-1} F_{KN,0} u'(C_0^R) = 0\end{aligned} \tag{35}$$

$$[k_0] : \theta_0 - \beta \mathbb{E}_0 \theta_1 (1 + F_{K,1} - \delta) = 0 \tag{36}$$

Steady state in the non-stochastic case Consider the special case in which there exists a $T \geqslant 0$ for which $g_t = g$ for all $t \geqslant T$, i.e. no more uncertainties after period T. Assume that there exists a solution to the Ramsey problem and that it converges to a time-invariant allocation, so that C, N, k are constant after some time. Then the steady state version of equation (31) implies:

$$1 = \beta(1 + F_K - \delta)$$

while the non-arbitrage condition for capital is $1 = \beta(1 + (1-\tau^K)F_K - \delta)$, so the optimal capital tax in the long run is zero. Indeed it is not a surprising result if we look at Judd [1985], where the agents are divided into two class. Capitalists do not work and workers do not save. The result of this extreme case shows that the long-run capital tax should be zero even if the government only consider the workers.

Ex-ante capital tax in the stochastic case We consider the capital tax that is not contingent on the realization of current state but is already set in the previous period. We define the ex-ante capital tax:

$$\bar{\tau}^K_{t+1} = \frac{E_t p^g_t(g_{t+1})\tau^K_{t+1} r_{t+1}}{E_t p^g_t(g_{t+1}) r_{t+1}}$$

To study the ex-ante capital tax in a stationary equilibrium, we now assume that the process $\{g_t\}$ follows a Markov process with transition probabilities $\pi(g'|g) = Prob(g_{t+1} = g'|g_t = g)$. An economy converges to a stationary if the stochastic process $\{g_t, k_t\}$ is a stationary, ergodic Markov process on the compact set $\mathcal{G} \times \mathcal{K}$ and the allocations can be described by time-invariant rule $C(g,k), n(g,k), k'(g,k)$.

Propositon Let $P^\infty(\cdot)$ be the probability measure over the outcomes of the stationary equilibrium. If there exists a stationary Ramsey equilibrium allocation, the ex-ante capital tax rate satisfies $P^\infty(\tau^K > 0) > 0$ and $P^\infty(\tau^K < 0) > 0$

Proof By the definition of ex ante capital tax:

$$\bar{\tau}^K_{t+1} \geqslant (\leqslant) 0 \Longleftrightarrow \sum_{g_{t+1}} p^g_t(g_{t+1}|g_t)\tau^K_{t+1} r_{t+1} \geqslant (\leqslant) 0 \Longleftrightarrow \sum_{g_{t+1}} p^g_t(g_{t+1}|g_t)[r_{t+1} + 1 - \delta] \leqslant (\geqslant) 0$$
$$\Longleftrightarrow u'(C^R_t) \leqslant (\geqslant) E_t \beta u'(C^R_{t+1})[1 + F_{K_{t+1}} - \delta] \tag{37}$$

From the first order condition of equation (29) and (30), we can solve

$$-\theta_t = \frac{(1-\alpha)u'(C^R_t) + \phi[u''(C^R_t)C^R_t + u'(C^R_t)] - \mu_t v'(N^K_t)u''(C^R_t)}{1-\lambda}$$

Define

$$H_t \equiv \frac{-\theta_t}{u'(C^R_t)} = \frac{1 - \alpha + \phi(1-\gamma_C)}{1-\lambda} - \frac{\mu_t v'(N^K_t)u''(C^R_t)}{(1-\lambda)u'(C^R_t)} \tag{38}$$

Then Equation (31) could be rewritten as:

$$u'(C_t^R)H_t = \beta E_t u'(C_{t+1}^R)H_{t+1}F_{K_{t+1}} \tag{39}$$

From the last equivalent condition of equation (37), we can get

$$H_t \geqslant (\leqslant)\frac{E_t\omega_{t+1}H_{t+1}}{E_t\omega_{t+1}} \tag{40}$$

where $\omega_{t+1} \equiv u'(C_{t+1}^R)(1 + F_{K,t+1} - \delta)$

Since a stationary Ramsey equilibrium has time-invariant allocation rule for C, N, k, equation (37) could be rewritten as:

$$\begin{aligned}\bar{\tau}(g_t, k_t) \geqslant (\leqslant)0 \Longleftrightarrow H(g_t, k_t) \geqslant (\leqslant)&\frac{\sum_{g_{t+1}} \pi(g_{t+1}|g_t)\omega_{t+1}(g_{t+1}, k'(g_t, k_t))H_{t+1}(g_{t+1}, k'(g_t, k_t))}{\sum_{g_{t+1}} \pi(g_{t+1}|g_t)\omega_{t+1}(g_{t+1}, k'(g_t, k_t))}\\ &\equiv \Gamma H(g_t, k_t)\end{aligned} \tag{41}$$

Here the operator Γ is a weighted average of H with the property that $\Gamma H^* = H^*$ for any constant H^*. Under some regularity conditions, $H(g_t, k_t)$ attains its maximum H^+ and minimum H^- in the stationary equilibrium. Follow Zhu [1992] proof, there must exist a constant H^* such that $\Gamma H^* = H^*$

We can find that H consists of two part: the first part is a constant which is identical to that in the representative agent economy and implies zero long-run capital tax with probability 1; the second part comes from the equilibrium condition of same proportional labor tax for the agents again, which makes the ex-ante capital tax in the stationary economy fluctuate around 0. So if the planner are allowed to levy agent-specific proportional labor tax, the ex-ante long-run capital tax would be zero.

6 Conclusion

When two types of agents co-exist in the economy, homogeneous labor tax rate imposes an additional constraint to the government, apart from the implementability and resources constraints. Then the optimal tax prescription of constant labor tax and long-run 0 capital tax does not hold any more, which mirrors the classical result that incomplete tax system overturns the uniform commodity taxation.

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